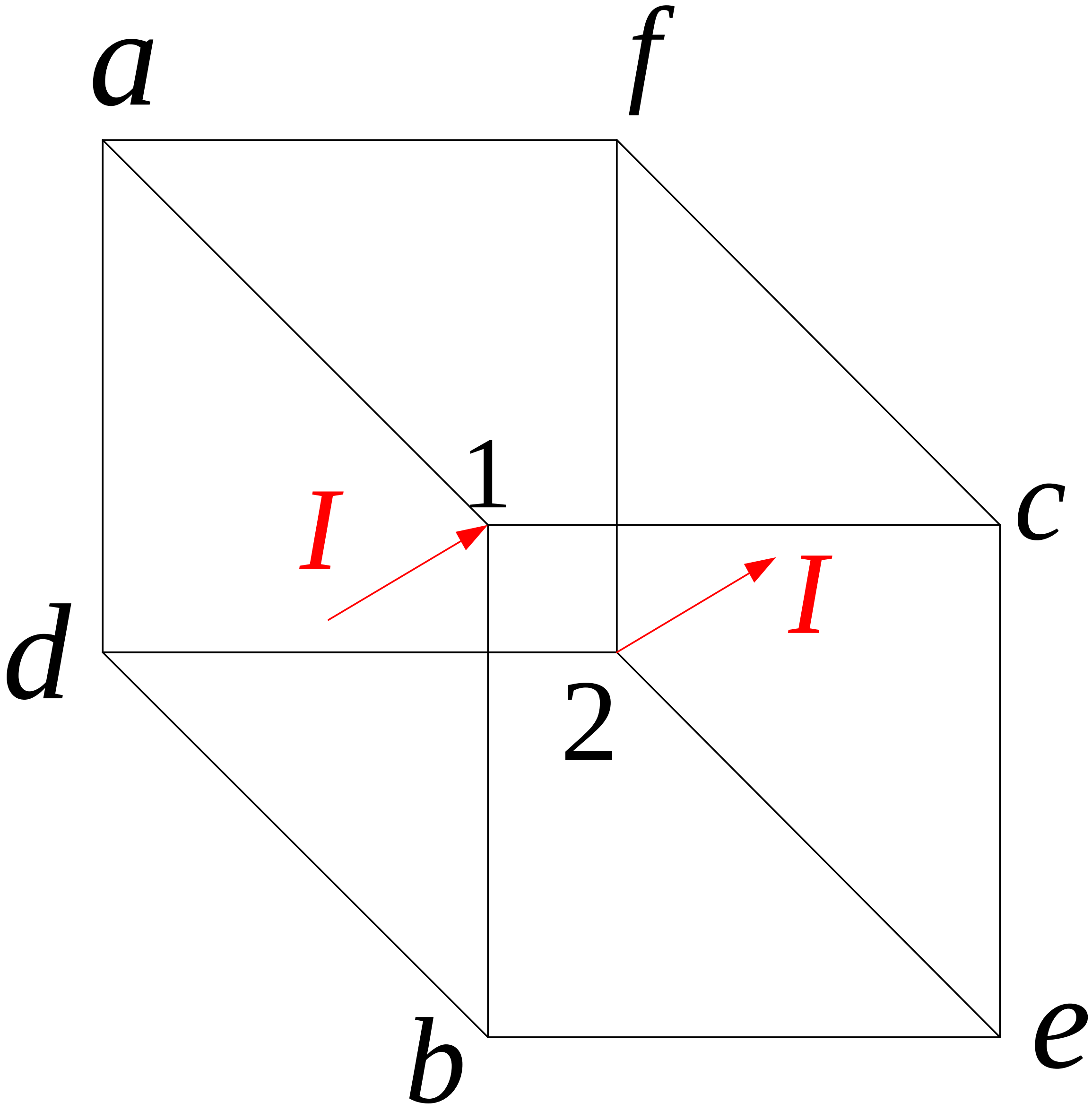
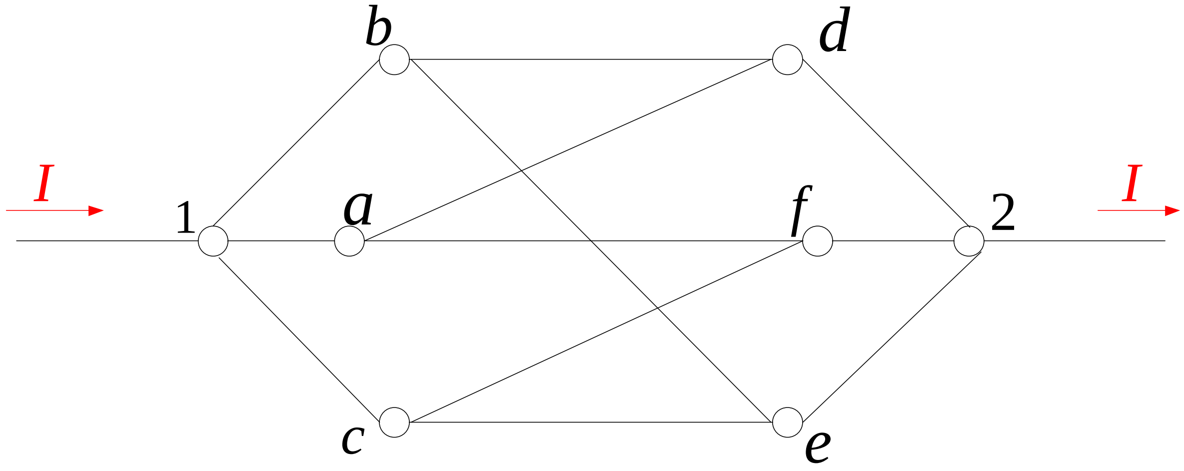
Lösung zur Übung 1.2.2/1





$$\left.\begin{matrix} a, b, c \\ d, e, f \end{matrix}\right\}$$ Äquipotentiallinien

Widerstand von:

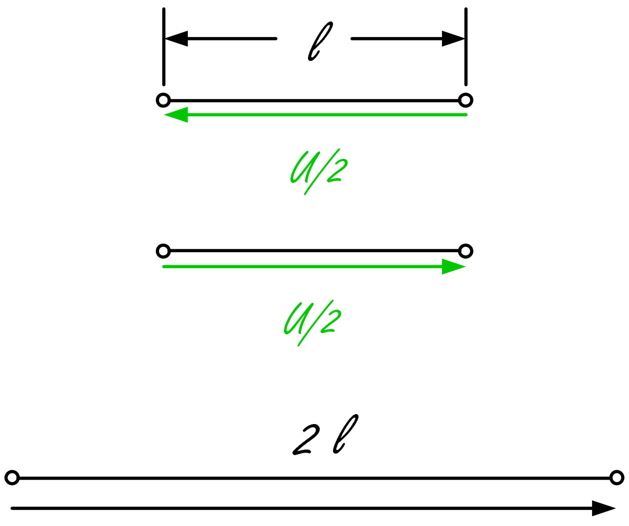
$$1 \text{ nach } a, b, c = \frac{1}{3}\,\Omega$$

$$2 \text{ nach } d, e, f = \frac{1}{3}\,\Omega$$

$$a, b, c \text{ nach } d, e, f = \frac{1}{6}\,\Omega$$

$$R_{ges} = \left(\frac{1}{3} + \frac{1}{3} + \frac{1}{6}\right)\Omega = \underline{\underline{\frac{5}{6}\,\Omega}}$$

Lösung zur Übung 1 . 2 . 3 / 1



a) $J = \frac{I}{A}$ $\quad I = \frac{U}{R}$ $\quad R = \frac{2 \cdot \ell \cdot \rho}{A}$

$$J = \frac{U}{A \cdot R} = \frac{U}{A \cdot \frac{2 \cdot \ell \cdot \rho}{A}} = \frac{90\,V}{2 \cdot 2 \cdot 10^2\, m \cdot 0{,}0178 \frac{V\,mm^2}{m}}$$

$$= 12{,}64 \frac{A}{mm^2}$$

b) $J = \frac{1}{A \cdot R} \cdot U$

$$\frac{dJ}{J} = \frac{1}{A \cdot R} \cdot \frac{dU}{\frac{U}{A \cdot R}}$$

$$\frac{dJ}{J} = \frac{dU}{U} = -20\,\%$$

Lösung zur Übung 1.2.4/1

a) $\rho = \frac{1}{\chi}$

$$R_{cu} = \frac{\ell}{\chi_{cu} \cdot A_{cu}} = \frac{\ell}{\chi_{Al} \cdot A_{Al}} = R_{Al}$$

$$\frac{\ell}{\chi_{cu} \cdot A_{cu}} = \frac{\ell}{\chi_{Al} \cdot A_{Al}}$$

$$A_{Al} = A_{cu} \cdot \frac{\chi_{cu}}{\chi_{Al}} = 10\ mm^2 \cdot \frac{56}{35} = 16\ mm^2$$

b)

$$R_2 = R_1[\,1 + \alpha_M\ (\vartheta_2 - \vartheta_1)\,]$$

$$R_2 = R_{20}[\,1 + \alpha_{20}\ (\vartheta_2 - 20\,^0C)\,]$$

$$-10\,^0C: \quad R_{-10} = R_{20}[\,1 + 4‰\ (-\,30)\,] = R_{20} \cdot 0{,}88$$

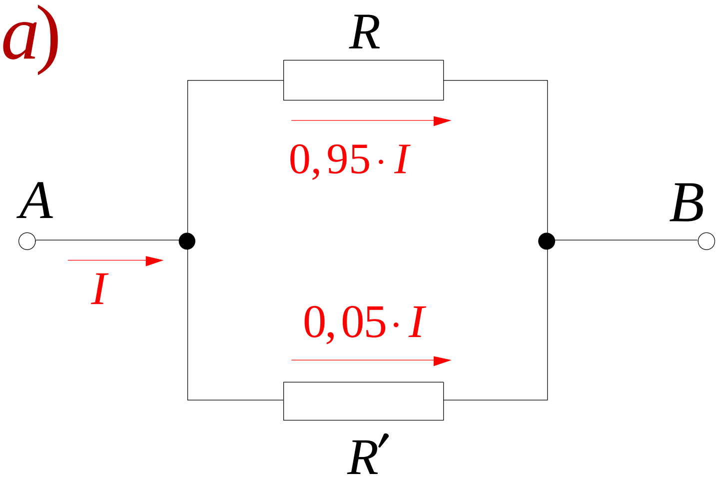
$$+\,30\,^0C: \quad R_{+30} = R_{20}[\,1 + 4‰\ (10)\,] = R_{20} \cdot 1{,}04$$

$$f = \frac{R_{+30} - R_{-10}}{R_{+30}} = \frac{R_{20} \cdot 1{,}04 - R_{20} \cdot 0{,}88}{R_{20} \cdot 1{,}04}$$

$$= \frac{1{,}04 - 0{,}88}{1{,}04} = 15{,}38\ \%$$

Lösung zur Übung 2.1.2/1

a)



$$U = (1 - p) \cdot I \cdot R = p \cdot I \cdot R'$$

$$R' = \frac{R \cdot (1 - p)}{p} = \frac{R \cdot 0{,}95}{0{,}05} = 19\ R$$

b)

$$R_{ges} = \frac{R \cdot R'}{R + R'} = \frac{19 \cdot R^2}{20 \cdot R} = \frac{19}{20} R$$

1. $R_{AB1} = R$

2. $R_{AB2} = R_{ges} = \frac{19}{20} R$

$$\frac{\Delta R_{AB}}{R_{AB}} = \frac{R_{AB2} - R_{AB1}}{R_{AB1}} = \frac{\frac{19}{20} R - R}{R}$$

$$= \frac{-\frac{1}{20} R}{R} = -\frac{1}{20} = -5\ \%$$

$$U_{AB} = I \cdot R_{AB}$$

$$\frac{dU_{AB}}{U_{AB}} = I \cdot \frac{dR_{AB}}{I \cdot R_{AB}}$$

$$\frac{dU_{AB}}{U_{AB}} = \frac{dR_{AB}}{R_{AB}} = -5\ \%$$

wird um 5 % kleiner

Lösung zur Übung 2.1.2/2

a)

$$I = \frac{U_0}{R_i + R_1 + R_2}$$

$$\alpha = \frac{U_2}{U_1} = \frac{I \cdot R_2}{I \cdot R_1} = \frac{R_2}{R_1}$$

$$\frac{d\alpha}{\alpha} = \frac{1}{R_1} \cdot \frac{dR_2}{\frac{R_2}{R_1}}$$

$$\frac{d\alpha}{\alpha} = \frac{dR_2}{R_2} = +2\ \%$$

b) $\alpha = \frac{R_2}{R_1}$

$$\frac{d\alpha}{\alpha} = \frac{-R_2}{\frac{R_2}{R_1}} \cdot \frac{dR_1}{R_1^2}$$

$$\frac{d\alpha}{\alpha} = -\frac{dR_1}{R_1} = \underline{\underline{-2\ \%}}$$

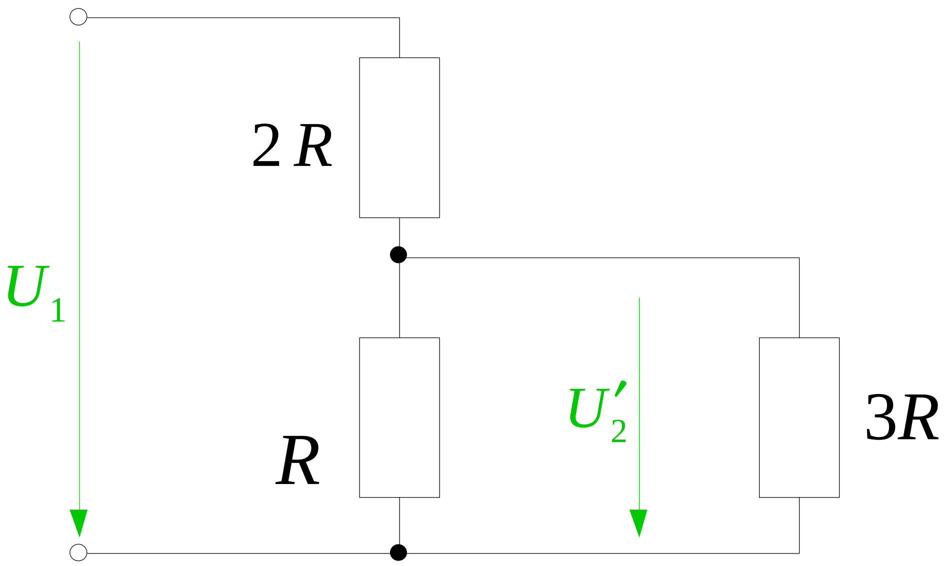
Näherung durch Tangentenbildung: exakter Wert der $\frac{1}{R_1}$ *–Funktion* $(-1{,}96\ \%)$

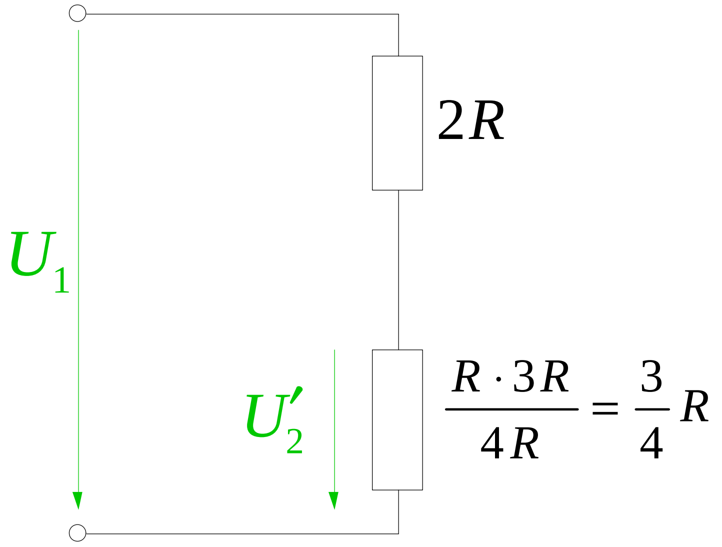
c) $\alpha = \frac{R_2}{R_1}$

$$\frac{d\alpha}{\alpha} = \frac{-R_2}{\frac{R_2}{R_1}} \cdot \frac{dR_1}{R_1^2} + \frac{dR_2}{R_1 \frac{R_2}{R_1}} = -\frac{dR_1}{R_1} + \frac{dR_2}{R_2}$$

$$= -2\ \% + 2\ \% = \underline{\underline{0}}$$

Lösung zur Übung 2.1.2/3





$$\frac{U_1}{U_2'} = \frac{2R + \frac{3}{4}R}{\frac{3}{4}R} = \frac{11}{4} \cdot \frac{4}{3} = \frac{11}{3}$$

$$\frac{U_2}{U_2'} = \frac{R}{3R} = \frac{1}{3}$$

$$\frac{U_1}{U_2} = \frac{U_1}{U_2'} \cdot \frac{U_2'}{U_2} = \frac{11}{3} \cdot \frac{3}{1} = \underline{\underline{11}}$$

Lösung zur Übung 2.1.2/4

$$\frac{U_1}{U_2} = \frac{U_1}{U} \cdot \frac{U}{U_2}$$

$$R' = R + R_x \qquad R'' = \frac{R_x \cdot R'}{R_x + R'} = \frac{R_x(R + R_x)}{R + 2R_x}$$

$$\frac{U}{U_2} = \frac{R'}{R_x} = \frac{R + R_x}{R_x} = 1 + \frac{R}{R_x}$$

$$\frac{U_1}{U} = \frac{R + R''}{R''} = 1 + \frac{R}{R''} = 1 + \frac{R(R + 2R_x)}{(R + R_x)R_x}$$

$$\frac{U_1}{U_2} = \left(1 + \frac{R(R + 2R_x)}{(R + R_x)R_x}\right)\left(1 + \frac{R}{R_x}\right)$$

$$\frac{U_1}{U_2} = \left[\frac{(R + R_x)R_x + R(R + 2R_x)}{\cancel{(R + R_x)}R_x}\right]\cdot\left(\frac{\cancel{R + R_x}}{R_x}\right)$$

$$= \frac{R_x^2 + R_x \cdot 3R + R^2}{R_x^2} = 10$$

$$9R_x^2 - R_x \cdot 3R - R^2 = 0$$

$$R_x^2 + \frac{1}{3}R_x \cdot R - \frac{R^2}{9} = 0$$

$$R_{x_{1,2}} = \frac{1}{6}R \pm \sqrt{\underbrace{\frac{1}{36}R^2 + \frac{1}{9}R^2}_{\frac{5}{36}\cdot R^2}} = \frac{1}{6}R\left[1 \pm \sqrt{5}\right] = \underline{\underline{0,539R}}$$

nur positives R_x sinnvoll

Lösung zur Übung 2.1.2/5

$$\frac{I_1}{I_4} = \frac{I_1}{I_2}\cdot\frac{I_2}{I_3}\cdot\frac{I_3}{I_4}$$

$$I_4 \cdot \underbrace{R_8}_{6\,\Omega} = I_3 \cdot \frac{R_7 \cdot R_8}{R_7 + R_8} = I_3 \cdot \frac{3 \cdot 6}{9}\Omega = I_3 \cdot 2\Omega$$

$$\Rightarrow \quad \frac{I_3}{I_4} = \frac{6}{2} = \underset{\sim}{3}$$

$$R' = R_5 + R_6 + \frac{R_7 \cdot R_8}{R_7 + R_8} = 10\Omega + 2\Omega = 12\Omega$$

$$\frac{I_2}{I_3} = \frac{R'}{R_4//R'} = \frac{\cancel{R'}}{\frac{R_4 \cdot \cancel{R'}}{R_4 + R'}} = \frac{R_4 + R'}{R_4} = 1 + \frac{R'}{R_4}$$

$$\frac{I_2}{I_3} = 1 + \frac{12}{4} = \underset{\sim}{4}$$

$$R'' = R_2 + R_3 + \frac{R_4 \cdot R'}{R_4 + R'} = 10\Omega + \frac{4 \cdot 12}{16}\Omega = 13\Omega$$

$$\frac{I_1}{I_2} = \frac{R''}{R_1 // R''} = \frac{R''}{\frac{R_1 \cdot R''}{R_1 + R''}} = \frac{R_1 + R''}{R_1} = 1 + \frac{R''}{R_1}$$

$$\frac{I_1}{I_2} = 1 + \frac{13}{2} = \frac{15}{2}$$

$$\frac{I_1}{I_4} = \frac{15}{2} \cdot 4 \cdot 3 = \underline{\underline{90}}$$

Lösung zur Übung 2.2.3/1

$$\sum U = 0 \qquad -I_1 R_1 + U_1 - I_1 R_2 + U_2 - I_3 R_3 = 0 \qquad (I)$$

$$I_5 (R_4 + R_5) - I_1 (R_1 + R_2) + U_1 = 0 \qquad (II)$$

$$\sum I = 0 \qquad I_1 - I_3 + I_5 = 0 \quad (III)$$

$$\Rightarrow \quad I_1 = I_3 - I_5$$

(III) in (I) $$(I_3 - I_5)(R_1 + R_2) + I_3 R_3 = U_2 + U_1$$

(III) in (II) $$(I_3 - I_5)(R_1 + R_2) - I_5 (R_4 + R_5) = U_1$$

$$I_3\left(R_1+R_2+R_3\right)-I_5\left(R_1+R_2\right)=U_2+U_1 \quad / \cdot \left(R_1+R_2\right)$$

$$-\left[I_3\left(R_1+R_2\right)-I_5\left(R_1+R_2+R_4+R_5\right)=U_1 \quad / \cdot \left(R_1+R_2+R_3\right)\right]$$

$$+I_5\left[-\left(R_1+R_2\right)^2+\left(R_1+R_2+R_4+R_5\right)\left(R_1+R_2+R_3\right)\right]$$

$$=\left(U_1+U_2\right)\left(R_1+R_2\right)-U_1\left(R_1+R_2+R_3\right)$$

$$+I_5\left[-\left(R_1+R_2\right)^2+\left(R_1+R_2\right)^2+\left(R_1+R_2+R_4+R_5\right)R_3+\left(R_1+R_2\right)\left(R_4+R_5\right)\right]$$

$$=-U_1R_3+U_2\left(R_1+R_2\right)$$

$$I_5\left[\left(R_1+R_2\right)\left(R_3+R_4+R_5\right)+R_3\left(R_4+R_5\right)\right]=-U_1R_3+U_2\left(R_1+R_2\right)$$

$$I_5=\frac{-U_1R_3+U_2\left(R_1+R_2\right)}{\left[\left(R_1+R_2\right)\left(R_3+R_4+R_5\right)+R_3\left(R_4+R_5\right)\right]}$$

Lösung zur Übung 2 . 2 . 3 / 2

$\sum U=0 \qquad I_1R_1-U_1-I_2R_2+U_2=0 \qquad (I)$

$I_2R_2-I_3\left(R_3+R_4\right)-U_3-U_2=0 \qquad (II)$

$$\Sigma I = 0 \qquad I_1 + I_2 + I_3 = 0 \Rightarrow I_1 = -I_2 - I_3 \qquad (III)$$

$$(III) \text{ in } (I) \qquad \left(-I_2 - I_3\right) R_1 - I_2 R_2 = U_1 - U_2 \qquad (IV)$$

$$(III) \text{ in } (II) \qquad I_2 R_2 - I_3 \left(R_3 + R_4\right) = U_2 + U_3 \qquad (V) / \cdot R_1$$

$$(IV) \quad - \left[-I_2 \left(R_1 + R_2\right) - I_3 R_1 = U_1 - U_2 \qquad / \cdot \left(R_3 + R_4\right)\right]$$

$$I_2 \left[R_1 R_2 + \left(R_1 + R_2\right)\left(R_3 + R_4\right)\right]$$

$$= \left(U_2 + U_3\right) R_1 - \left(U_1 - U_2\right)\left(R_3 + R_4\right)$$

$$I_2 = \frac{-U_1\left(R_3 + R_4\right) + U_2\left(R_1 + R_3 + R_4\right) + U_3 R_1}{R_1\left(R_2 + R_3 + R_4\right) + R_2\left(R_3 + R_4\right)}$$

Lösung zur Übung 2.2.3/3

$$\Sigma U = 0 \qquad -I_1 R_1 + U_1 - I_2 R_2 = 0 \qquad (I)$$

$$I_2 R_2 - I_5 R_5 + U_2 - I_3 R_3 = 0 \qquad (II)$$

$$I_3 R_3 - U_2 + I_4 R_4 = 0 \qquad (III)$$

$$\Sigma I = 0 \qquad -I_1 + I_2 + I_5 = 0 \qquad (IV) \qquad \Rightarrow I_1 = I_2 + I_5$$

$$-I_5 + I_3 - I_4 = 0 \qquad (V) \qquad \Rightarrow I_3 = I_4 + I_5$$

$(IV)\,in(I)\quad -(I_2+I_5)R_1+U_1-I_2R_2=0 \quad\Rightarrow\quad I_2(R_1+R_2)+I_5R_1=U_1$

$(V)\,in(II)\quad I_2R_2-I_5R_5+U_2-(I_4+I_5)R_3=0\Rightarrow\quad -I_2R_2+I_4R_3+I_5(R_3+R_5)=U_2$

$(V)\,in(III)\quad (I_4+I_5)R_3-U_2+I_4R_4=0 \quad\Rightarrow\quad I_4(R_3+R_4)+I_5R_3=U_2$

$$I_2(R_1+R_2) \qquad\qquad +I_5R_1 = U_1 \quad (VI) \qquad /\cdot R_2$$

$$-I_2R_2 \qquad +I_4R_3 \qquad +I_5(R_3+R_5) = U_2 \quad (VII) \quad /\cdot(R_1+R_2)$$

$$+I_4(R_3+R_4) \quad +I_5R_3 = U_2 \quad (VIII) \qquad /\cdot R_3\cdot(R_1+R_2)$$

$(VI)+(VII)\quad I_4R_3(R_1+R_2)+I_5[R_1R_2+(R_1+R_2)(R_3+R_5)]$

$=U_1\cdot R_2+U_2(R_1+R_2) \qquad /\cdot(R_3+R_4)\quad$ (IX)

(IX) - $(VIII)$ $\quad I_5\{(R_3+R_4)[R_1R_2+(R_1+R_2)(R_3+R_5)]-R_3^2(R_1+R_2)\}$

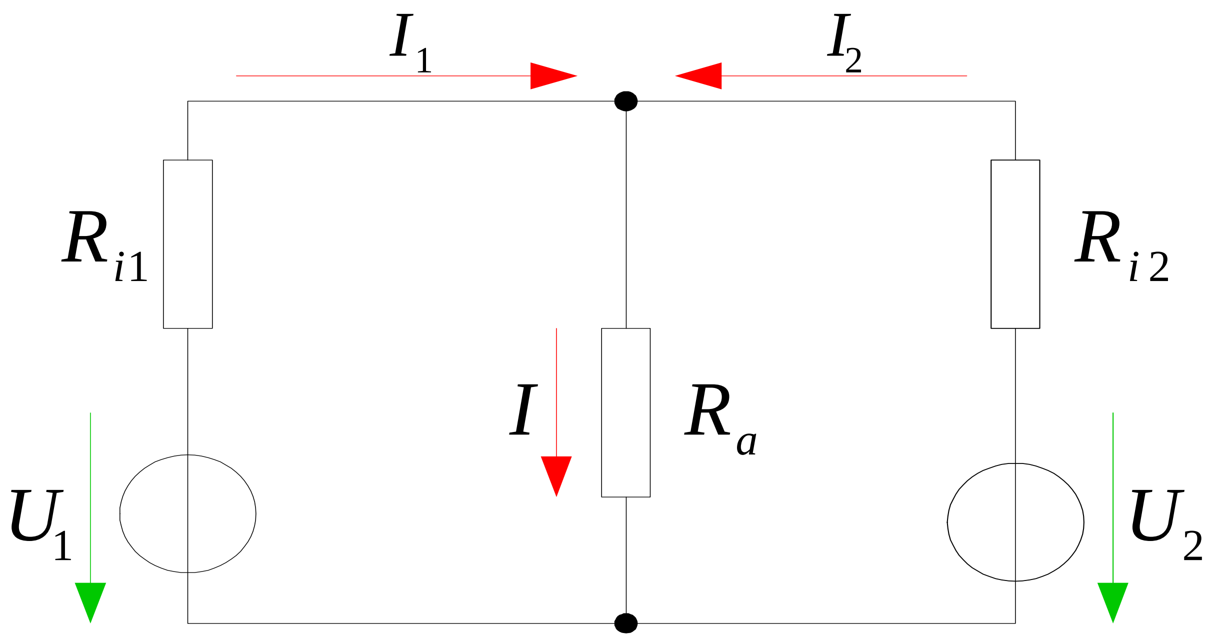
$=U_1\cdot R_2(R_3+R_4)+U_2(R_1+R_2)(R_3+R_4)-U_2R_3(R_1+R_2)$

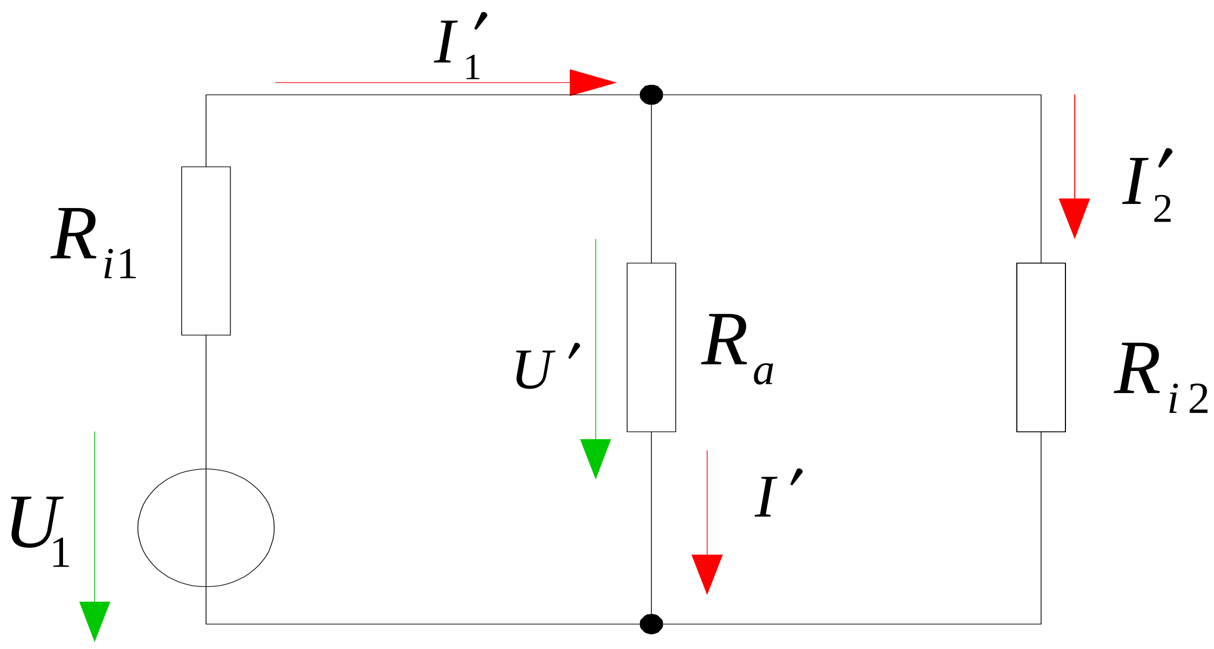
$I_5[(R_3+R_4)(R_1R_2+R_1R_5+R_2R_5)+(R_3+R_4)(R_1R_3+R_2R_3)-R_3^2(R_1+R_2)]$

$=U_1\cdot R_2(R_3+R_4)+U_2R_4(R_1+R_2)$

$$I_5=\frac{U_1\cdot R_2(R_3+R_4)+U_2R_4(R_1+R_2)}{(R_3+R_4)(R_1R_2+R_1R_5+R_2R_5)+R_3R_4(R_1+R_2)}$$

Lösung zur Übung 2 . 3 / 1


I1
I2
Ri1
Ri2
I
Ra
U1
U2


I1′
I2′
Ri1
U′
Ra
Ri2
I′
U1

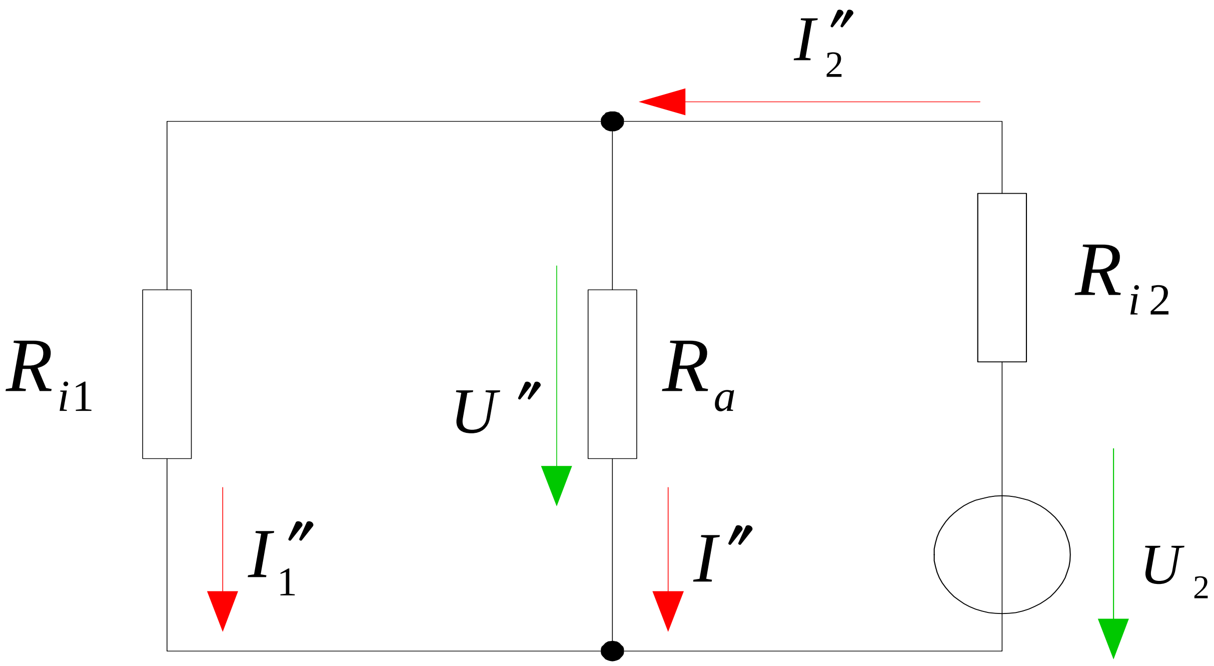
$$R_p' = \frac{R_a \cdot R_{i2}}{R_a + R_{i2}} = \frac{1 \cdot 0,01}{1,01} = 9,901 \cdot 10^{-3}\Omega$$

$$I_1' = \frac{U_1}{R_{i1} + R_p'} = \frac{10V}{(0,1 + 9,901 \cdot 10^{-3})\Omega} = 90,991A$$

$$U' = I_1' \cdot R_p' = 90,991A \cdot 9,901 \cdot 10^{-3}\Omega = 0,9009V$$

$$I_2' = \frac{U'}{R_{i2}} = 90,09A$$

$$I' = \frac{U'}{R_a} = 0,9009A$$



$$R_p'' = \frac{R_a \cdot R_{i1}}{R_a + R_{i1}} = \frac{1 \cdot 0,1}{1,1} = 0,09091\Omega$$

$$I_2'' = \frac{U_2}{R_{i2} + R_p''} = \frac{10V}{(0,01 + 0,09091)\Omega} = 99,098A$$

$$U'' = I_2'' \cdot R_p'' = 99,098A \cdot 0,09091\Omega = 9,008999V$$

$$I_1'' = \frac{U''}{R_{i1}} = 90,09A$$

$$I'' = \frac{U''}{R_a} = 9,008A$$

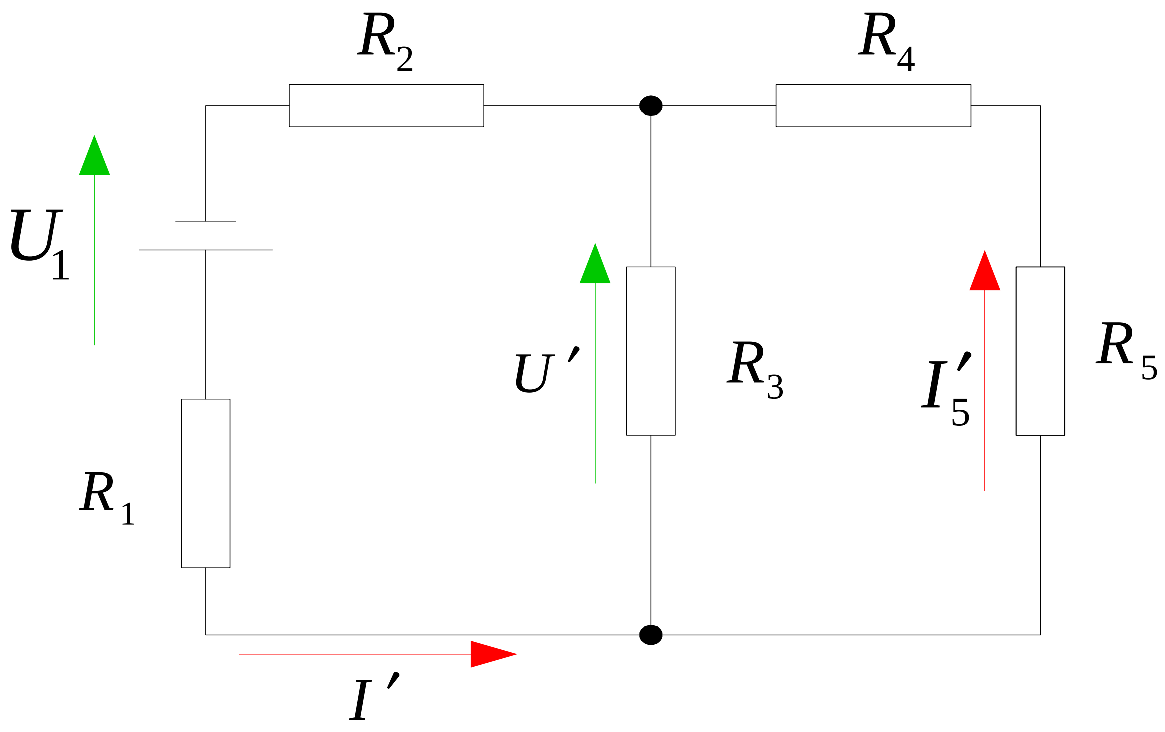
$I = I' + I'' = (0,9009 + 9,008)A = \underline{\underline{9,9089A}}$ *näherungsweise erfüllt*

$I_1 = I_1' - I_1'' = (90,991 - 90,09)A = \underline{\underline{0,901A}} < 4A$ *realisierbar*

$I_2 = -I_2' + I_2'' = (-90,09 + 99,098)A = \underline{\underline{9,008A}} > 6A$ *überlastet*

Nicht realisierbar, da Netzgerät II überlastet

Lösung zur Übung 2 . 3 / 2



$$R'_p = \frac{R_3(R_4 + R_5)}{R_3 + R_4 + R_5}$$

$$I' = \frac{U_1}{R_1 + R_2 + R'_p}$$

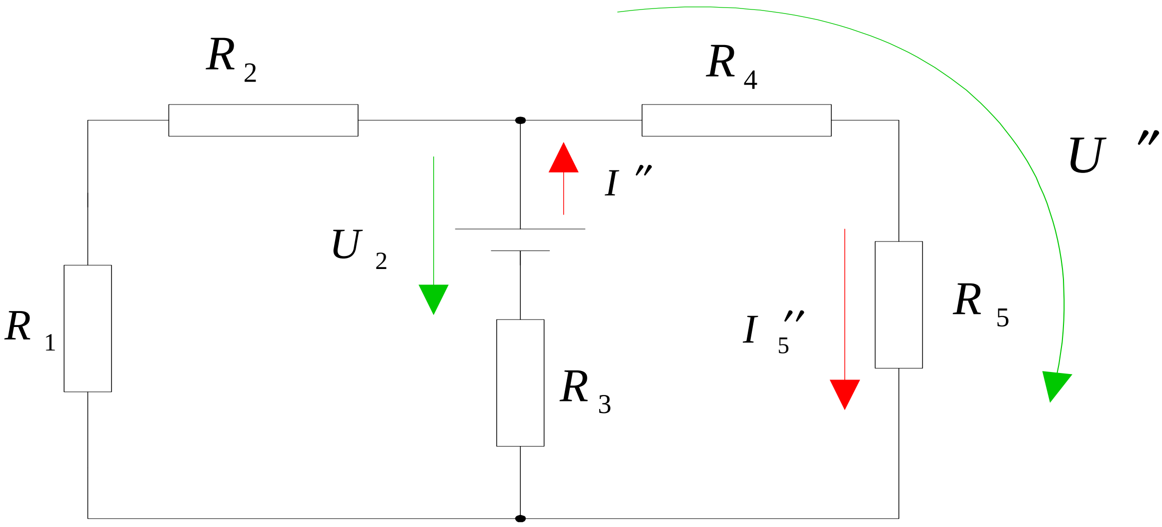
$$U' = I' \cdot R'_p$$

$$I'_5 = \frac{U'}{R_4 + R_5} = \frac{I' \cdot R'_p}{R_4 + R_5}$$

$$I'_5 = \frac{U_1}{(R_1 + R_2 + R'_p)} \cdot \frac{R'_p}{(R_4 + R_5)}$$

$$I'_5 = \frac{U_1}{[R_1 + R_2 + \frac{R_3(R_4 + R_5)}{R_3 + R_4 + R_5}]} \cdot \frac{R_3 (R_4 + R_5)}{(R_3 + R_4 + R_5)(R_4 + R_5)}$$

$$I'_5 = \frac{U_1 \cdot R_3}{(R_1 + R_2)(R_3 + R_4 + R_5) + R_3(R_4 + R_5)}$$



$$R_p'' = \frac{(R_1 + R_2)(R_4 + R_5)}{R_1 + R_2 + R_4 + R_5}$$

$$I'' = \frac{U_2}{R_3 + R_p''}$$

$$U'' = I'' \cdot R_p''$$

$$I_5'' = \frac{U''}{R_4 + R_5} = \frac{I'' \cdot R_p''}{R_4 + R_5}$$

$$I_5'' = \frac{U_2}{(R_3 + R_p'')} \cdot \frac{R_p''}{(R_4 + R_5)}$$

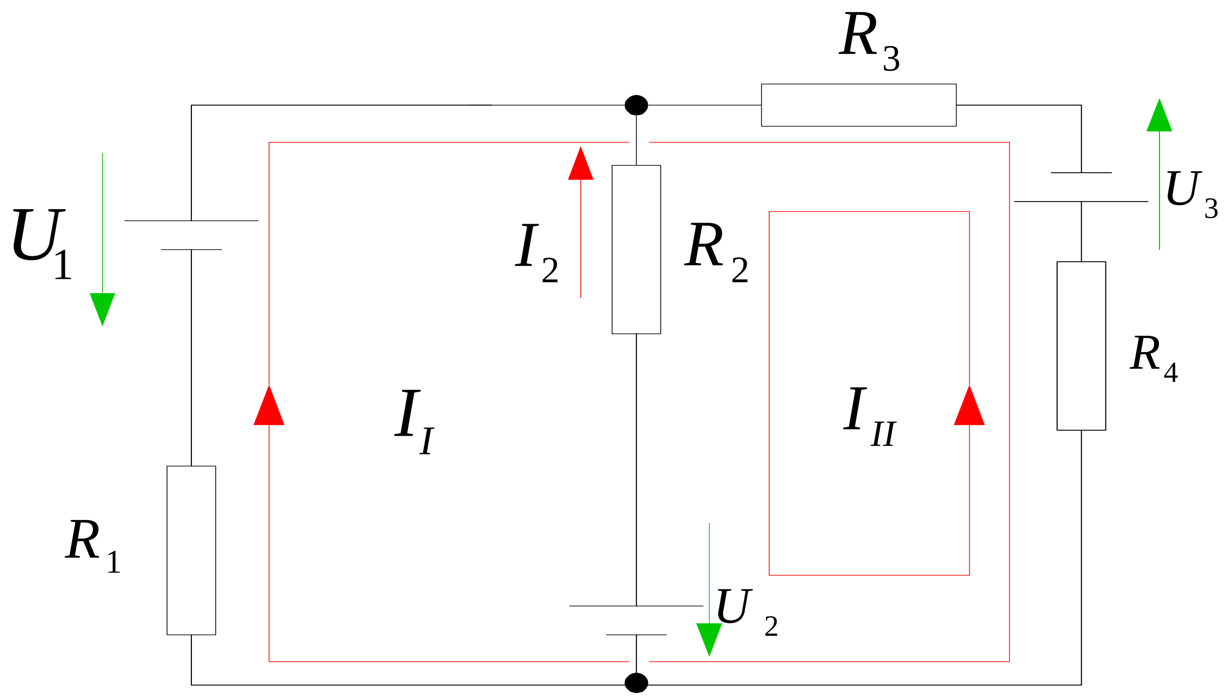
$$I_5'' = \frac{U_2}{\left[R_3 + \frac{(R_1 + R_2)(R_4 + R_5)}{R_1 + R_2 + R_4 + R_5}\right]} \cdot \frac{(R_1 + R_2)\,(R_4 + R_5)}{(R_1 + R_2 + R_4 + R_5)\,(R_4 + R_5)}$$

$$I_5'' = \frac{U_2 \cdot (R_1 + R_2)}{R_3(R_1 + R_2 + R_4 + R_5) + (R_1 + R_2)(R_4 + R_5)}$$

$$I_5'' = \frac{U_2 \cdot (R_1 + R_2)}{(R_1 + R_2)(R_3 + R_4 + R_5) + R_3(R_4 + R_5)}$$

$$I_5 = I_5' - I_5'' = \underline{\underline{\frac{U_1 \cdot R_3 - U_2 \cdot (R_1 + R_2)}{(R_1 + R_2)(R_3 + R_4 + R_5) + R_3(R_4 + R_5)}}}$$

Lösung zur Übung 2 . 4 / 1



$$I_I(R_1+R_3+R_4)-I_{II}(R_3+R_4)-U_1-U_3=0 \quad (I)$$

$$-I_I(R_3+R_4)+I_{II}(R_2+R_3+R_4)+U_3+U_2=0 \quad (II)$$

$$\text{aus } (II) \quad I_I=\frac{U_2+U_3+I_{II}(R_2+R_3+R_4)}{R_3+R_4} \quad (III)$$

$$(III) \text{ in } (I) \quad \left[\frac{U_2+U_3+I_{II}(R_2+R_3+R_4)}{R_3+R_4}\right](R_1+R_3+R_4)-I_{II}(R_3+R_4)$$

$$=U_1+U_3$$

$$I_{II}[(R_2+R_3+R_4)(R_1+R_3+R_4)-(R_3+R_4)^2]$$

$$=(U_1+U_3)(R_3+R_4)-(U_2+U_3)(R_1+R_3+R_4)$$

$$I_{II}[R_1(R_2+R_3+R_4)+R_2(R_3+R_4)+\cancel{(R_3+R_4)^2}-\cancel{(R_3+R_4)^2}]$$

$$=U_1(R_3+R_4)-U_2(R_1+R_3+R_4)-U_3(R_1+\cancel{R_3}+\cancel{R_4}-\cancel{R_3}-\cancel{R_4})$$

$$I_{II}=\frac{U_1(R_3+R_4)-U_2(R_1+R_3+R_4)-U_3R_1}{R_1(R_2+R_3+R_4)+R_2(R_3+R_4)}$$

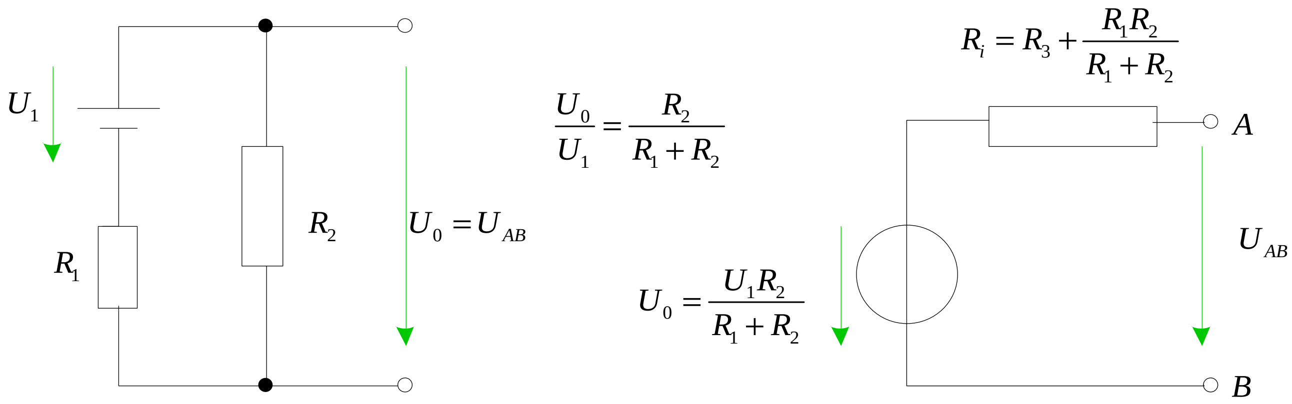
$$\underline{\underline{I_2=-I_{II}}}$$

Lösung zur Übung 2.5.5/1

Innenwiderstand: $R_i = R_{AB} = R_3 + \frac{R_1 \cdot R_2}{R_1 + R_2}$

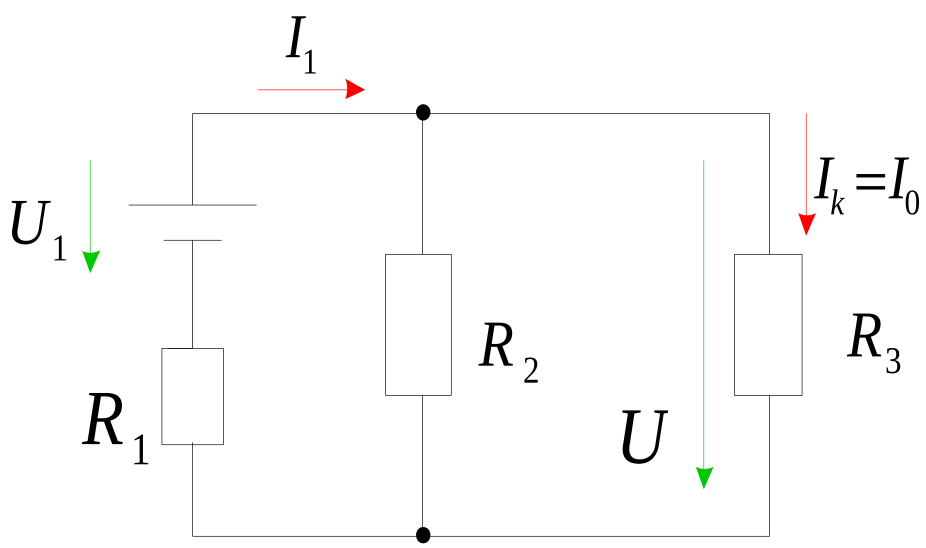
Leerlaufspg.: $U_L = U_0 = U_{AB}$

durch R_3 fließt im Leerlauf kein Strom

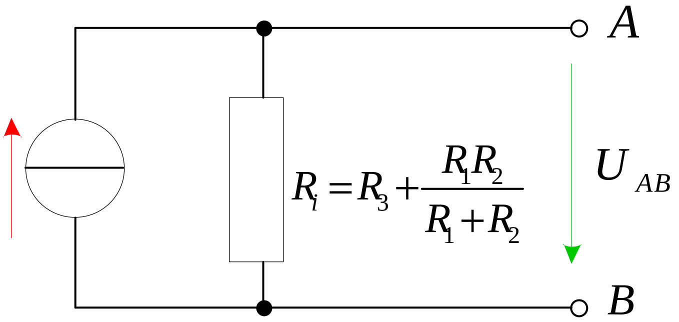


Berechnung für Stromquelle:

Kurzschlussstrom $I_k = I_0$ (Klemmen A - B kurzgeschlossen)



$$I_0 = \frac{U_1 R_2}{R_1(R_2+R_3)+R_2+R_3}$$



$$R_i = R_3 + \frac{R_1 R_2}{R_1+R_2}$$

$$I_1 = \frac{U_1}{R_1 + \dfrac{R_2 R_3}{R_2 + R_3}}$$

$$U = I_1 \cdot \frac{R_2 R_3}{R_2 + R_3} = \frac{U_1}{R_1 + \dfrac{R_2 R_3}{R_2 + R_3}} \cdot \frac{R_2 R_3}{R_2 + R_3} = \frac{U_1 \cdot R_2 R_3}{R_1(R_2 + R_3) + R_2 R_3}$$

$$I_0 = \frac{U}{R_3} = \frac{U_1 \cdot R_2}{R_1(R_2 + R_3) + R_2 R_3}$$

Lösung zur Übung 2.5.5/2

a) *Innenwiderstand* $R_i = R_{AB} = \dfrac{R_2(R_1+R_3)}{R_1+R_2+R_3}$

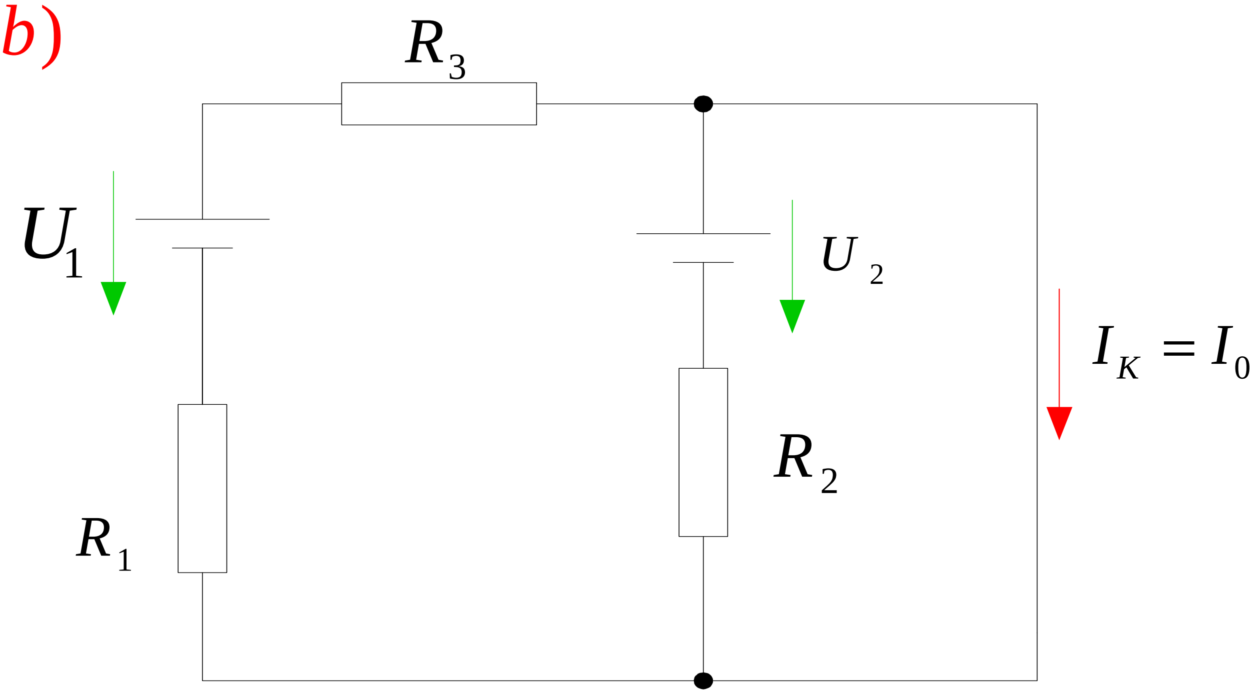
Leerlaufspg. $U_L = U_0 = U_{AB}$

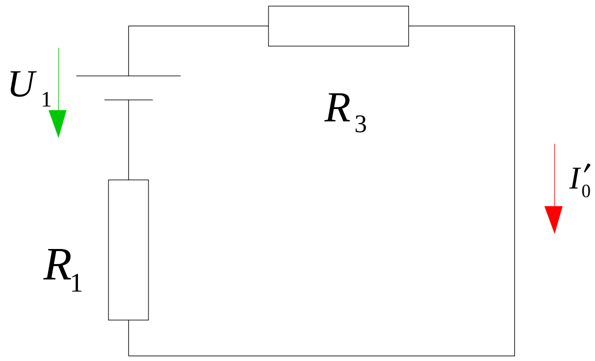
$$\sum U = 0 \quad (R_1+R_3)I - U_1 + U_2 + I\cdot R_2 = 0 \;\Rightarrow\; I = \frac{U_1 - U_2}{R_1+R_2+R_3} \quad \text{(I)}$$

$$-I\cdot R_2 - U_2 + U_{AB} = 0 \;\Rightarrow\; U_0 = U_{AB} = U_2 + I\cdot R_2 = U_2 + \frac{R_2(U_1-U_2)}{R_1+R_2+R_3}$$

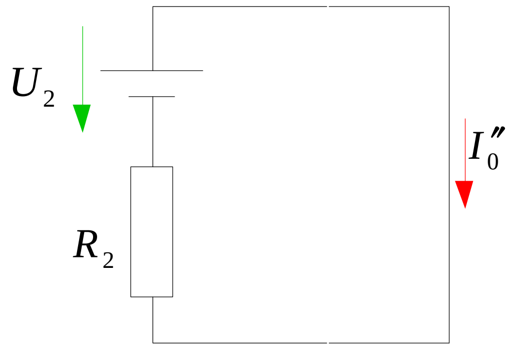
$$U_0 = \frac{U_2(R_1 + \cancel{R_2} + R_3) + R_2(U_1 - \cancel{U_2})}{R_1+R_2+R_3} = \underline{\underline{\frac{U_1 R_2 + U_2(R_1+R_3)}{R_1+R_2+R_3}}}$$

b)





$$I_0' = \frac{U_1}{R_1 + R_3}$$

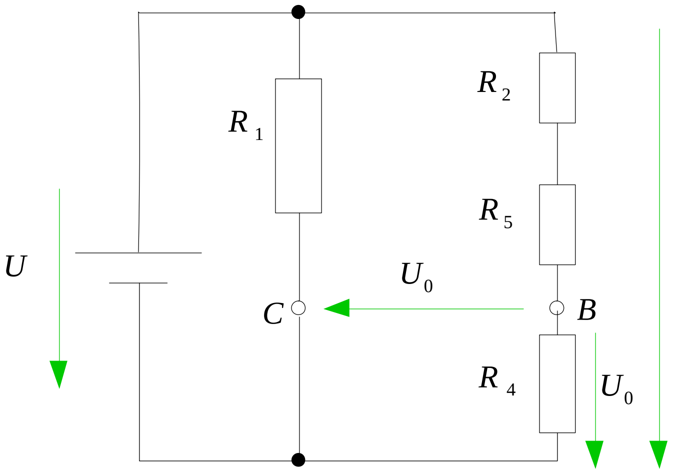


$$I_0'' = \frac{U_2}{R_2}$$

$$I_K = I_0 = I_0' + I_0'' = \frac{U_1}{R_1 + R_3} + \frac{U_2}{R_2} = \underline{\underline{\frac{U_1 R_2 + U_2 (R_1 + R_3)}{R_2 (R_1 + R_3)}}}$$

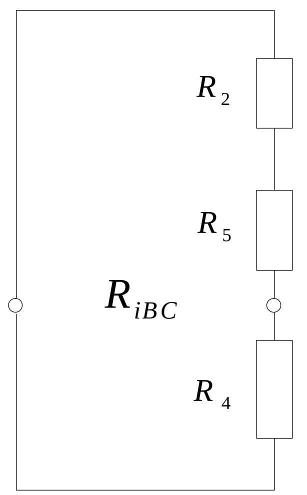
<u>Lösung zur Übung 2 . 5 . 5 / 3</u>

a) Ersatzspannungsquelle für B - C

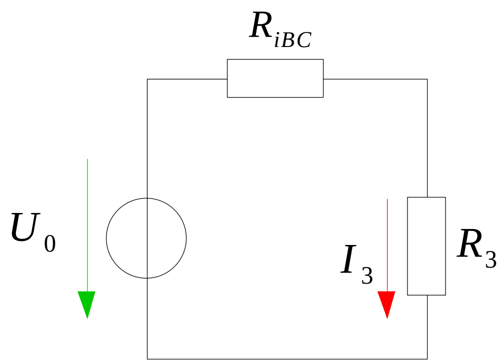


$$U_{BC} = U_L = U_0$$

$$\frac{U_0}{U} = \frac{R_4}{R_2 + R_4 + R_5}$$



$$R_{iBC} = \frac{R_4(R_2 + R_5)}{R_2 + R_4 + R_5}$$



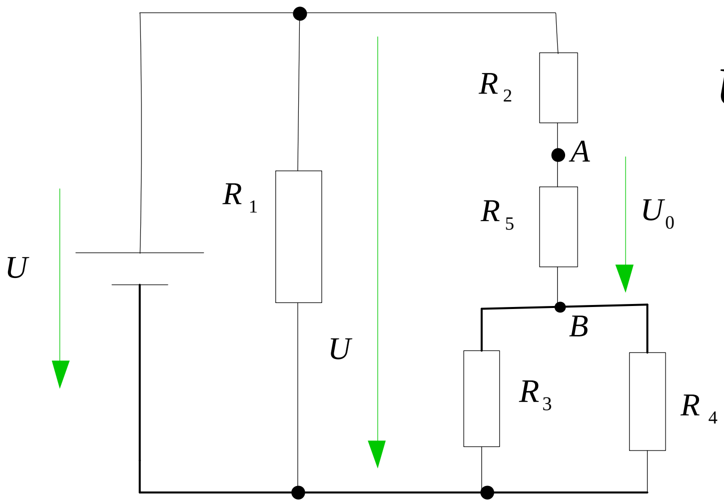
$$I_3 = \frac{U_0}{R_{iBC} + R_3} = \frac{U \cdot R_4}{(R_2 + R_4 + R_5)(R_{iBC} + R_3)}$$

$$I_3 = \frac{U \cdot R_4}{(R_2 + R_4 + R_5)\left(R_3 + \frac{R_4(R_2 + R_5)}{R_2 + R_4 + R_5}\right)}$$

$$I_3 = \frac{U \cdot R_4}{R_3(R_2 + R_4 + R_5) + R_4(R_2 + R_5)}$$

$$\underline{\underline{I_3 = \frac{U \cdot R_4}{(R_2 + R_5)(R_3 + R_4) + R_3 R_4}}}$$

b)



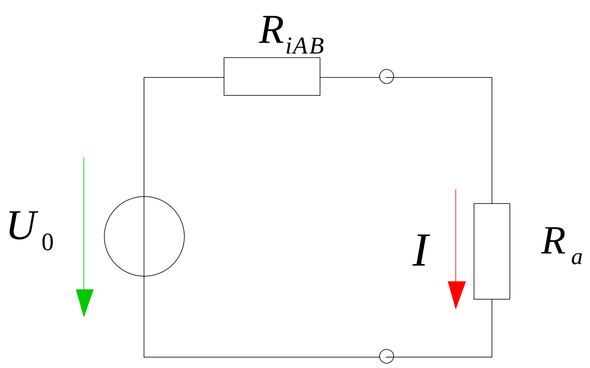
$$U_{AB} = U_L = U_0$$

$$\frac{U_0}{U} = \frac{R_5}{R_2 + R_5 + \frac{R_3 \cdot R_4}{R_3 + R_4}}$$

$$U_0 = \frac{U \cdot R_5 (R_3 + R_4)}{(R_2 + R_5)(R_3 + R_4) + R_3 R_4}$$

$$R_{iAB} = R_5 // [R_2 + (R_3 // R_4)] = \frac{R_5 (R_2 + \frac{R_3 R_4}{R_3 + R_4})}{R_5 + R_2 + \frac{R_3 R_4}{R_3 + R_4}} = \frac{R_5 [R_2 (R_3 + R_4) + R_3 R_4]}{(R_5 + R_2)(R_3 + R_4) + R_3 R_4} = R_a$$

R_1 durch Spannungsquelle kurzgeschlossen



$$I = \frac{U_0}{R_{iAB} + R_a} = \frac{U_0}{2 R_{iAB}}$$

$$= \frac{U \cdot R_5 (R_3 + R_4)}{2 R_5 [(R_2 (R_3 + R_4) + R_3 R_4)]}$$

Lösung zur Übung 2.5.5/4

3 Möglichkeiten

a) Reine Serienschaltung

$$U_{Lges} = N \cdot U_L = 72 \cdot 2V = 144V$$

$$R_{iges} = N \cdot R_i = 72 \cdot 0{,}5\Omega = 36\Omega$$

$$I = \frac{U_{Lges}}{R_a + R_{iges}} = \frac{144V}{(4+36)\Omega} = \underline{\underline{3{,}6\,A}}$$

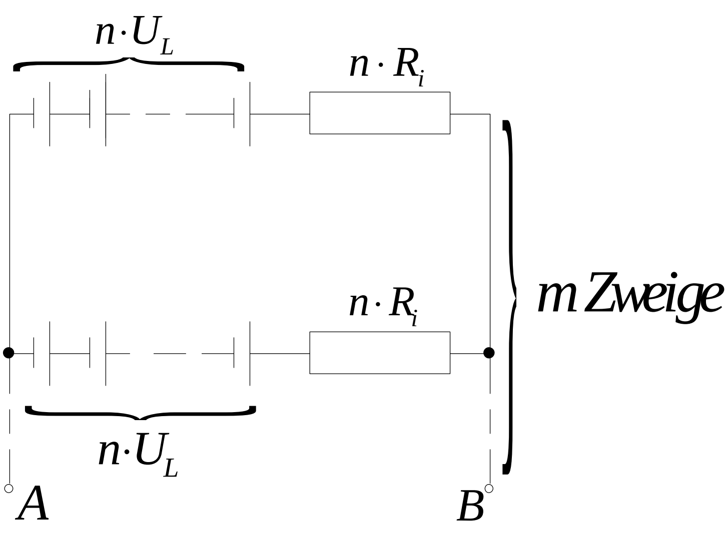
b) Reine Parallelschaltung

$$U_{Lges} = U_L = 2V$$

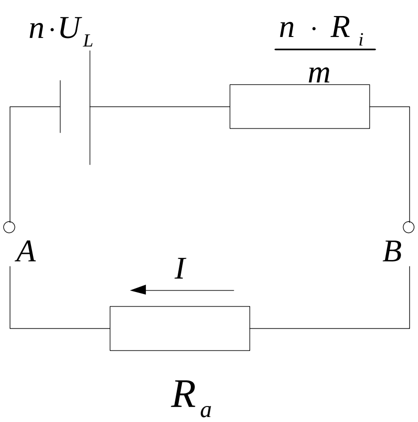
$$R_{iges} = \frac{R_i}{N} = \frac{0{,}5}{72}\Omega = 6{,}94\,m\Omega$$

$$I = \frac{U_{Lges}}{R_a + R_{iges}} = \frac{2V}{(4 + 6{,}94 \cdot 10^{-3})\Omega} = \underline{\underline{0{,}499\,A}}$$

c) Gemischte Schaltung



Ersatzspannungsquelle



$$R_{iAB} = \frac{n \cdot R_i}{m}$$ (m gleiche parallele Widerstände $n \cdot R_i$)

$$I = \frac{n \cdot U_L}{\frac{n \cdot R_i}{m} + R_a}$$

Gesamtzahl der Batterien ist $N = m \cdot n \quad \Rightarrow \quad m = \frac{N}{n}$

$U_L = const.$

Strom wird max., wenn Nenner min.

$$I = \frac{n \cdot U_L}{\frac{n^2 \cdot R_i}{N} + R_a} = \frac{U_L}{\frac{n^2 \cdot R_i + N \cdot R_a}{n \cdot N}}$$

$$\frac{d}{dn}\left(\frac{n^2 \cdot R_i + N \cdot R_a}{n \cdot N}\right) = \frac{2n \cdot R_i \cdot n \cdot N - N(n^2 \cdot R_i + N \cdot R_a)}{(n \cdot N)^2} = 0$$

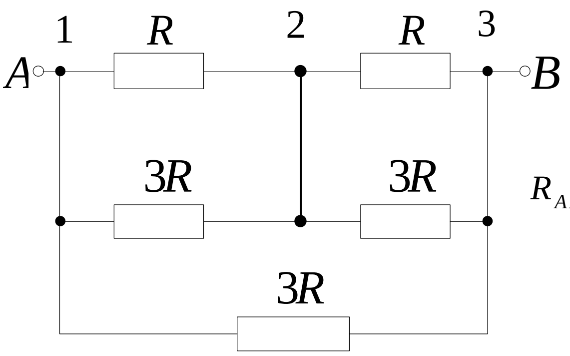
$$n^2 \cdot R_i \cdot N = N^2 \cdot R_a$$

$$n = \sqrt{\frac{N \cdot R_a}{R_i}} = \sqrt{\frac{72 \cdot 4}{0,5}} = \underline{\underline{24}} \qquad m = \frac{N}{n} = \frac{72}{24} = \underline{\underline{3}}$$

$$I = \frac{n \cdot U_L}{\frac{n \cdot R_i}{m} + R_a} = \frac{24 \cdot 2V}{\frac{24 \cdot 0,5\Omega}{3} + 4\Omega} = \frac{48V}{8\Omega} = \underline{\underline{6A}}$$

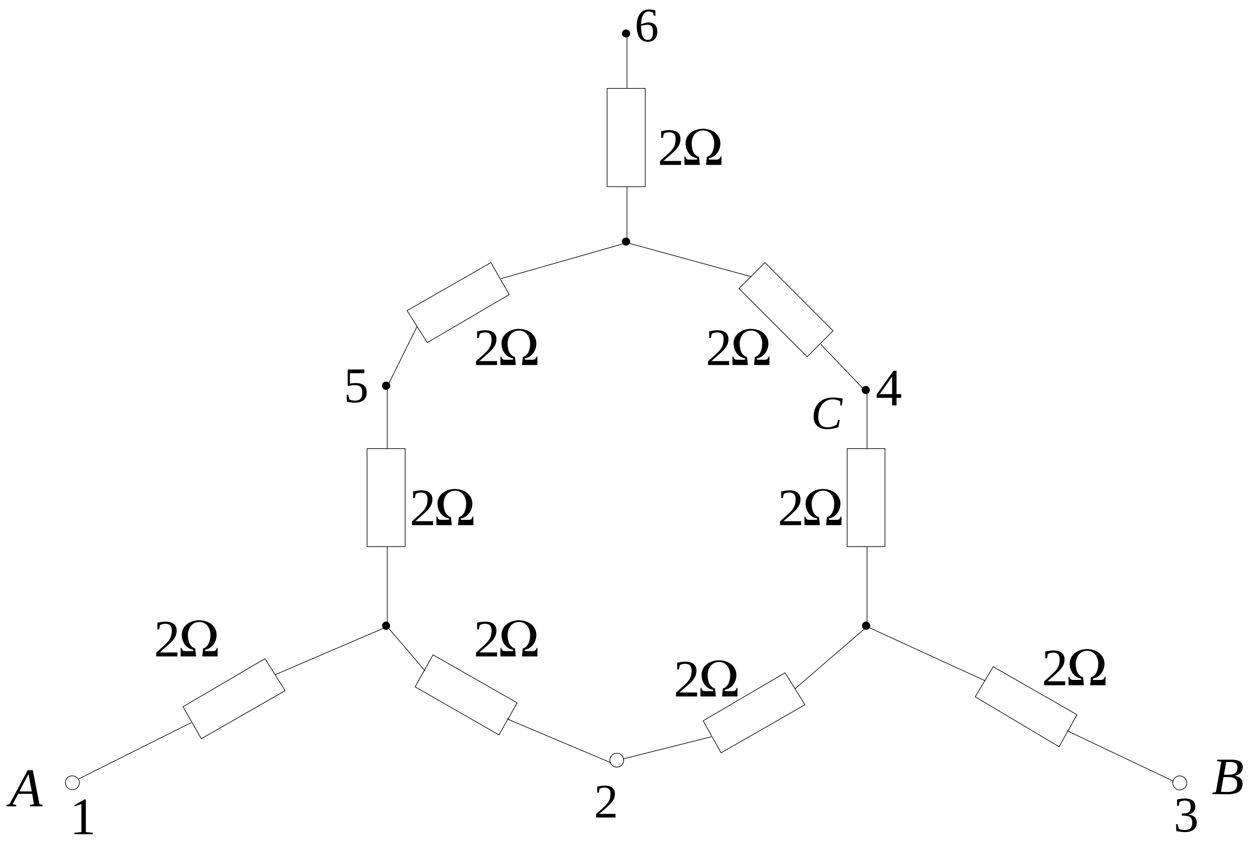
Lösung zur Übung 2.5.6/1

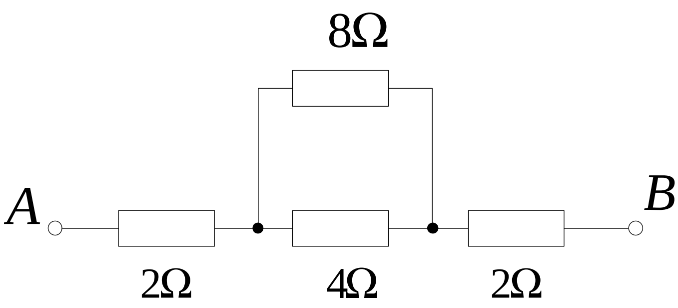
a)



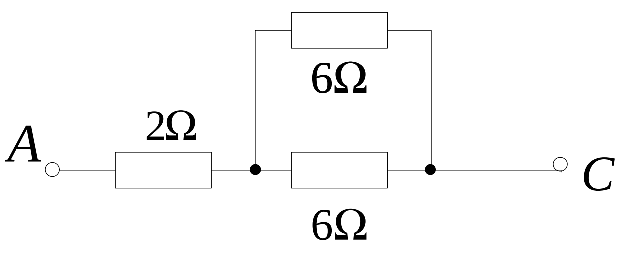
$$R_{AB} = \underbrace{\left(2\frac{R \cdot 3R}{R + 3R}\right)}_{\frac{3}{2}R} // 3R = \frac{\frac{3}{2}R \cdot 3R}{\frac{3}{2}R + 3R} = \frac{\frac{9}{2}R^2}{\frac{9}{2}R} = R$$

2. Weg: Punkte 0, 2 sind Äquipotentialpunkte (durch R(0-2) fließt kein Strom, R kann man herausnehmen) $\Rightarrow R_{AB} = \frac{R}{2} + \frac{R}{2} = R$





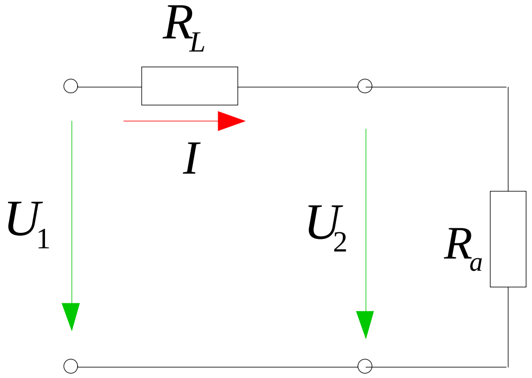
$$R_{AB} = 4\Omega + \frac{4\Omega \cdot 8\Omega}{4\Omega + 8\Omega} = \underline{\underline{6,67\Omega}}$$



$$R_{AC} = 2\Omega + 3\Omega = \underline{\underline{5\Omega}}$$

Lösung zur Übung 2.5.7.2/1

$$R_L = \frac{\rho \cdot 2 \cdot \ell}{A} = \frac{0{,}01786\Omega\, mm^2 \cdot 2 \cdot 10^5\, m}{150\, mm^2 \quad m} = 23{,}8\Omega$$



$$P_{R_a} = I^2 \cdot R_a \qquad P_{R_i} = I^2 \cdot R_L$$

$$\eta = \frac{P_{R_a}}{P_{R_a} + P_{R_i}} = \frac{R_a}{R_a + R_L} \Rightarrow R_a = \frac{R_L \cdot \eta}{1-\eta}$$

$$R_a = \frac{23{,}8\Omega \cdot 0{,}95}{1-0{,}95} = 452{,}2\Omega$$

$$I = \frac{U_1}{R_L + R_a} = \frac{100kV}{(23{,}8+452{,}2)\Omega} = 210{,}1A$$

$$P_{R_a} = I^2 \cdot R_a = \underline{\underline{19{,}96MW}}$$

b) $U_2 = U_1 - I \cdot R_L$

$$P_{R_a} = U_2 \cdot I = (U_1 - I \cdot R_L) \cdot I = U_1 \cdot I - I^2 \cdot R_L$$

$$\frac{dP_{R_a}}{dI} = U_1 - 2I \cdot R_L = 0 \Rightarrow I_{\max} = \frac{U_1}{2 \cdot R_L} = \frac{10^5 V}{2 \cdot 23{,}8\Omega} = 2{,}1kA$$

$$P_{R_{a\max}} = (U_1 - I_{\max} \cdot R_L) \cdot I_{\max}$$

$$= (10^5 V - 2{,}1 \cdot 10^3 A \cdot 2 \cdot 23{,}8\Omega) \cdot 2{,}1 \cdot 10^3 A = \underline{\underline{105MW}}$$

c) $P_{R_{a\max}} = I^2_{\max} \cdot R_a \Rightarrow R_a = \frac{P_{R_{a\max}}}{I^2_{\max}} = \frac{105 \cdot 10^6 W}{(2{,}1 \cdot 10^3 A)^2} = 23{,}8\Omega$

$$\eta = \frac{R_a}{R_a + R_L} = \frac{23{,}8\Omega}{23{,}8\Omega + 23{,}8\Omega} = 0{,}5 = \underline{\underline{50\ \%}}$$

Lösung zur Übung 4.3.1/1

a) $I = \iint_A \vec{J} \cdot d\vec{A} = J_X \cdot A = J_X \cdot a^2 \Rightarrow \underline{\underline{J_X = \frac{I}{a^2}}}$

unabhängig vom Material

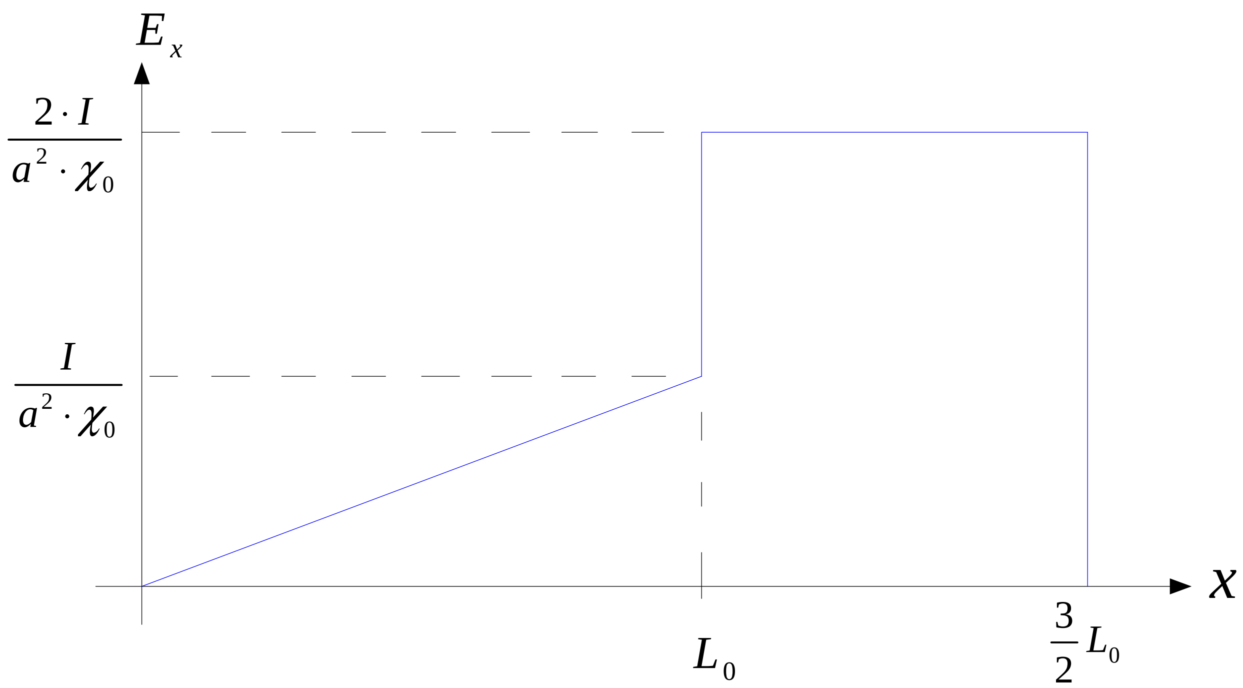
b) $\vec{J} = \chi \cdot \vec{E}$

$J_x = \chi \cdot E_x$

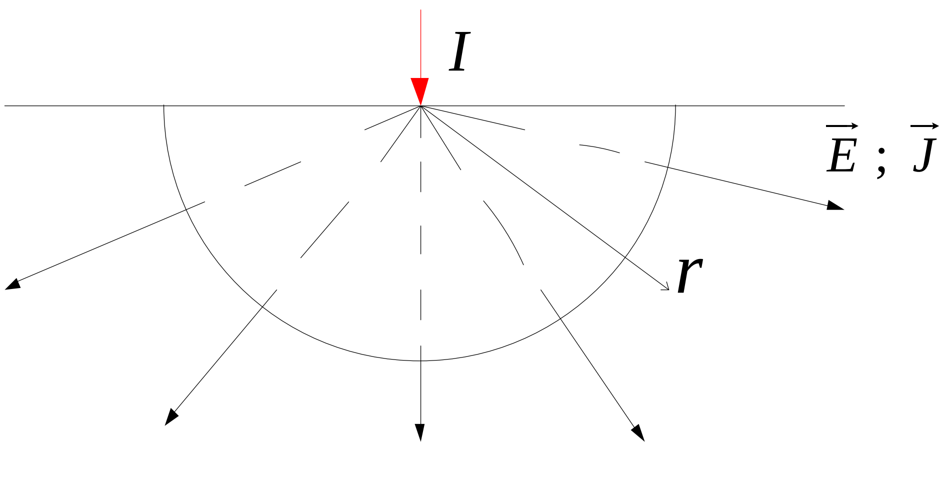
Teil 1: $E_{x_1} = \frac{J_x}{\chi_1} = \frac{I}{a^2} \cdot \frac{x}{\chi_0 \cdot L_0}$

$E_{x_1}(x = 0) = 0 \qquad E_{x_1}(x = L_0) = \frac{I}{a^2 \cdot \chi_0}$

Teil 2 $\quad E_{x_2} = \frac{J_x}{\chi_2} = \frac{I}{a^2} \cdot \frac{2}{\chi_0}$



Lösung zur Übung 4.3.1/2

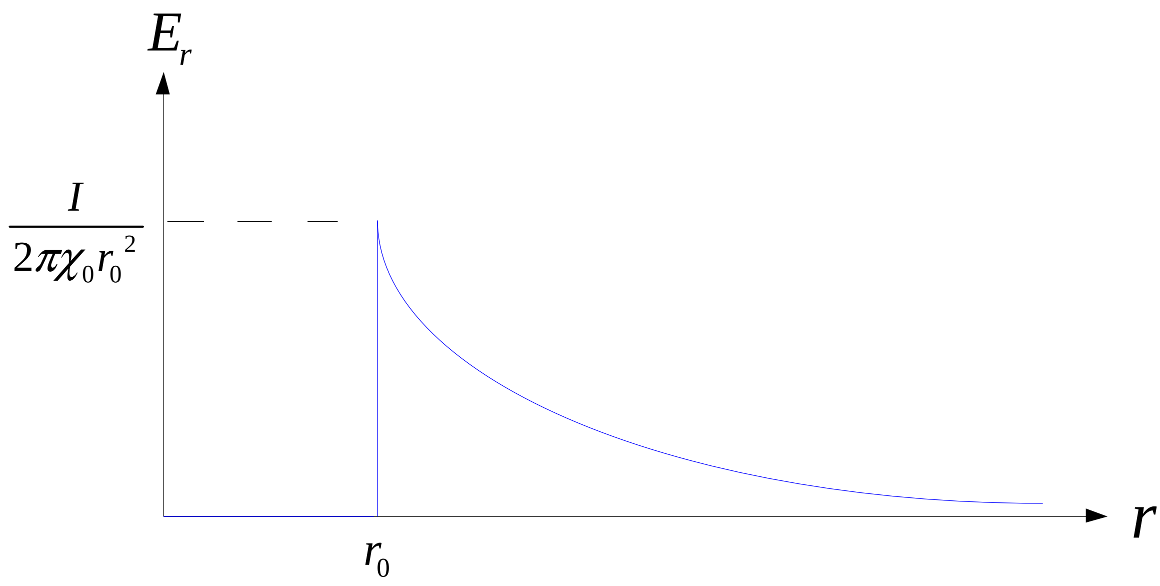


a) $I = \iint \vec{J} \cdot d\vec{A} = J_r \cdot \underbrace{\iint dA}_{2\pi r^2} = J_r \cdot 2\pi r^2$ (halbe Kugelfläche)

$J_r = \frac{I}{2\pi r^2}$ ($J_r \mathrel{\hat{=}} \vec{J}$ in radialer Richtung)

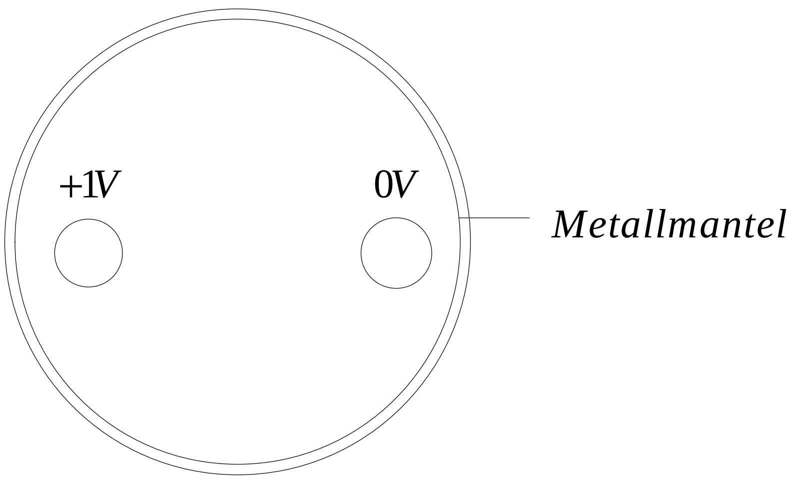
$\vec{J} = \chi \cdot \vec{E}$

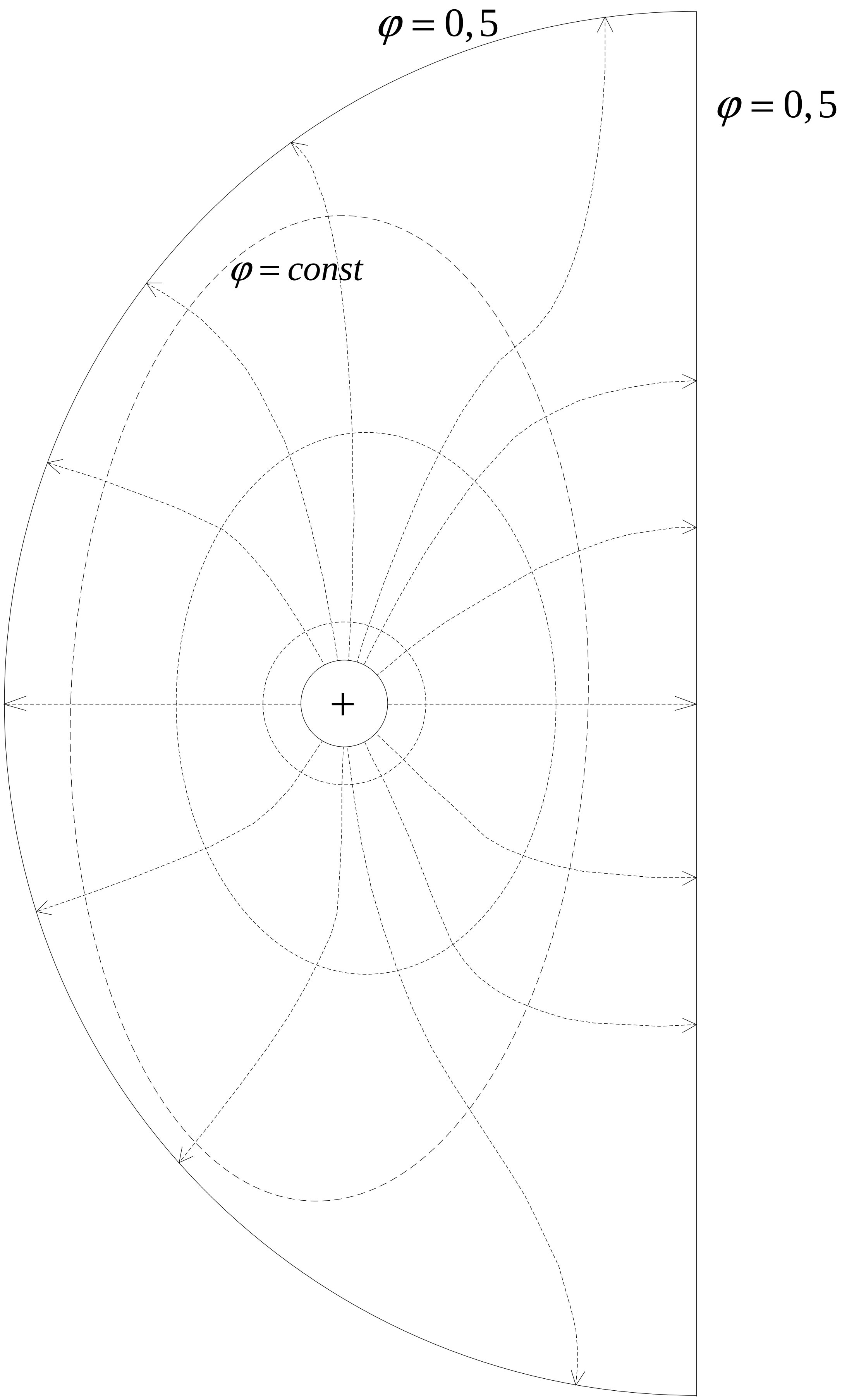
$E_r = \frac{J_r}{\chi_0} = \frac{I}{2\pi\chi_0 r^2}$ für $r > r_0$



b) $U_{12} = \int_1^2 \vec{E} \cdot d\vec{s} = \int_{r_1}^{r_2} E_r \cdot dr = \frac{I}{2\pi\chi_0} \int_{r_1}^{r_2} \underbrace{\frac{dr}{r^2}}_{-\frac{1}{r}} = \underline{\underline{\frac{I}{2\pi\chi_0}(\frac{1}{r_1} - \frac{1}{r_2})}}$

Lösung zur Übung 4.3.2/1




$\varphi = 0,5$
$\varphi = 0,5$
$\varphi = const$
+

Lösung zur Übung 4 . 3 . 3 / 1

a) $$U = \int \vec{E} \cdot d\vec{s} = \int_{R_1}^{R_2} E_1(r) \cdot dr + \int_{R_2}^{R_3} E_2(r) \cdot dr$$

$$I = \iint \vec{J} \cdot d\vec{A} = J \cdot \underbrace{\iint dA}_{2\pi r \cdot \ell} = J \cdot 2\pi r \cdot \ell$$

$$J = \frac{I}{2\pi r \cdot \ell}$$

$$\vec{J} = \chi \cdot \vec{E}$$

$$E_1 = \frac{J}{\chi_1} = \frac{I}{2\pi \chi_1 r \cdot \ell} \qquad E_2 = \frac{J}{\chi_2} = \frac{I}{2\pi \chi_2 r \cdot \ell}$$

$$U = \frac{I}{2\pi \ell}\left[\frac{1}{\chi_1}\int_{R_1}^{R_2} \underbrace{\frac{dr}{r}}_{\ln(r)} + \frac{1}{\chi_2}\int_{R_2}^{R_3} \underbrace{\frac{dr}{r}}_{\ln(r)}\right]$$

$$= \frac{I}{2\pi \ell}\left[\frac{1}{\chi_1}\ln\left(\frac{R_2}{R_1}\right) + \frac{1}{\chi_1}\ln\left(\frac{R_3}{R_2}\right)\right]$$

$$\Rightarrow \quad I = \frac{U \cdot 2\pi \ell}{\frac{1}{\chi_1}\ln\left(\frac{R_2}{R_1}\right) + \frac{1}{\chi_2}\ln\left(\frac{R_3}{R_2}\right)}$$

b) $$R = \frac{U}{I} \qquad E_1 = \frac{J}{\chi_1} = \frac{I}{2\pi \chi_1 r \cdot \ell} \qquad E_2 = \frac{J}{\chi_2} = \frac{I \; R_0}{2\pi \chi_0 r^2}$$

$$R = \frac{I}{2\pi \ell}\left[\frac{1}{\chi_1}\int_{R_1}^{R_2} \underbrace{\frac{dr}{r}}_{\ln(r)} + \frac{R_0}{\chi_0}\int_{R_2}^{R_3} \underbrace{\frac{dr}{r^2}}_{-\frac{1}{r}}\right]$$

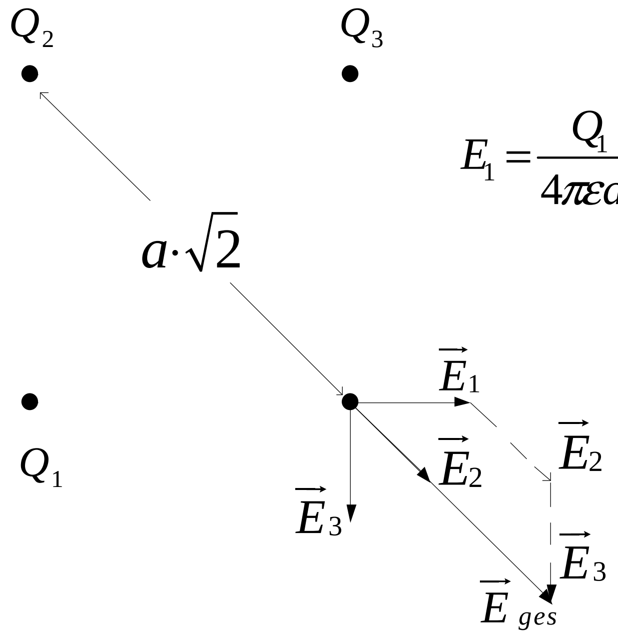
$$= \frac{I}{2\pi \ell}\left[\frac{1}{\chi_1}\ln\left(\frac{R_2}{R_1}\right) + \frac{R_0}{\chi_0}\left(\frac{1}{R_2} - \frac{1}{R_3}\right)\right]$$

Lösung zur Übung 4.4.3.2/1

a) $\vec{F} = Q \cdot \vec{E}$, wobei $\vec{E}$ diejenige elektr. Feldstärke ist, die im Punkt A durch Q_1, Q_2 und Q_3 verursacht wird.

Für eine Punktladung gilt: aus (4.4.3.1/3) $\vec{E} = \frac{Q}{4\pi\varepsilon r^2} \cdot \frac{\vec{r}}{r}$

$$E_1 = \frac{Q_1}{4\pi\varepsilon a^2} \qquad E_2 = \frac{Q_2}{4\pi\varepsilon (a\cdot\sqrt{2})^2} \qquad E_3 = \frac{Q_3}{4\pi\varepsilon a^2}$$



graphische Lösung

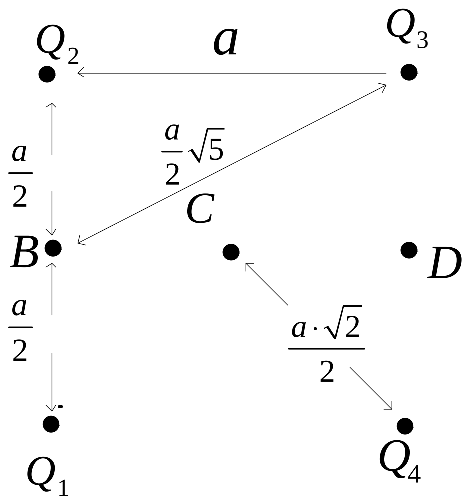
$$E_{ges} = E_2 + \sqrt{E_1^2 + E_3^2}$$

$$= \frac{Q}{4\pi\varepsilon 2a^2} + \sqrt{\frac{Q^2}{(4\pi\varepsilon a^2)^2} + \frac{Q^2}{(4\pi\varepsilon a^2)^2}}$$

$$E_{ges} = \frac{Q}{4\pi\varepsilon 2a^2} + \frac{Q}{4\pi\varepsilon a^2} \cdot \sqrt{2} = \frac{Q}{4\pi\varepsilon a^2}\left[\frac{1}{2} + \sqrt{2}\right]$$

$$F = \frac{Q_4 \cdot Q}{4\pi\varepsilon a^2}\left[\frac{1}{2} + \sqrt{2}\right]$$

b)



Potential einer Punktladung

aus (4.4.3.2 / 6)

$$\varphi = \frac{Q}{4\pi\varepsilon r}$$

$$\varphi_B = \varphi_D = \sum_{i=1}^{4}\varphi_i = \frac{Q_1}{4\pi\varepsilon\frac{a}{2}} + \frac{Q_2}{4\pi\varepsilon\frac{a}{2}} + \frac{Q_3}{4\pi\varepsilon\frac{a}{2}\sqrt{5}} + \frac{Q_4}{4\pi\varepsilon\frac{a}{2}\sqrt{5}}$$

$$\varphi_B = \varphi_D = \frac{Q\cdot 2}{4\pi\varepsilon a}\left[1+1+\frac{1}{\sqrt{5}}+\frac{1}{\sqrt{5}}\right] = \underline{\underline{\frac{Q}{\pi\varepsilon a}\left[1+\frac{1}{\sqrt{5}}\right]}}$$

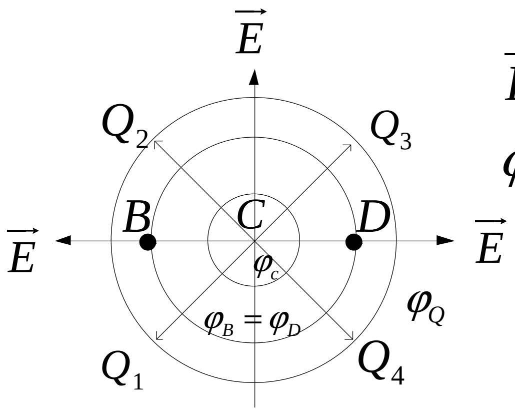
$$\varphi_C = \sum_{i=1}^{4}\varphi_i = \frac{Q_1}{4\pi\varepsilon\frac{a\sqrt{2}}{2}} + \frac{Q_2}{4\pi\varepsilon\frac{a\sqrt{2}}{2}} + \frac{Q_3}{4\pi\varepsilon\frac{a\sqrt{2}}{2}} + \frac{Q_4}{4\pi\varepsilon\frac{a\sqrt{2}}{2}}$$

$$\varphi_C = \frac{4\cdot Q\cdot 2\sqrt{2}}{4\pi\varepsilon\cdot a\sqrt{2}\sqrt{2}} = \underline{\underline{\frac{Q}{\pi\varepsilon a}\cdot\sqrt{2}}}$$

c)

$$U_{BC} = \varphi_B - \varphi_C = \frac{Q}{\pi\varepsilon a}\left[1+\frac{1}{\sqrt{5}}-\sqrt{2}\right] = \underline{\underline{0{,}033\cdot\frac{Q}{\pi\varepsilon a}}}$$

$$U_{BD} = \varphi_B - \varphi_D = 0$$

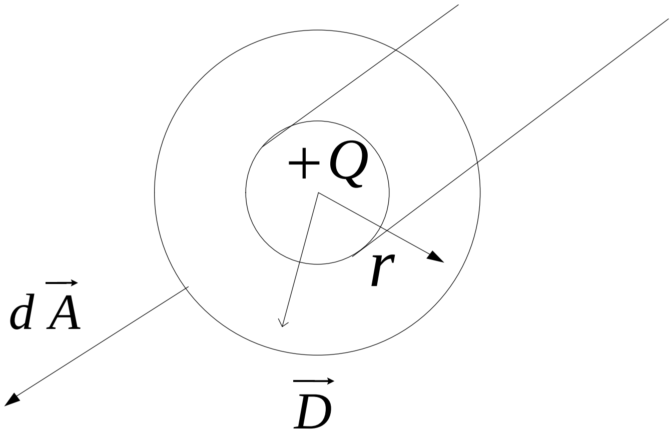


$$\vec{E}_{ges(C)} = 0$$

$$\varphi_C \neq 0$$

Lösung zur Übung 4 . 4 . 4 / 1

a)



$$Q = \oiint \vec{D} \cdot d\vec{A} = D_{(r)} \cdot 2\pi \cdot \ell \cdot r \quad \Rightarrow \quad D_{(r)} = \frac{Q}{2\pi \cdot \ell \cdot r}$$

$$E_{(r)} = \frac{D_{(r)}}{\varepsilon}$$

$$U_{(r)} = \int_{r_i}^{r_a} E_{(r)} \cdot dr = \int_{r_i}^{r_a} \frac{D_{(r)}}{\varepsilon} \cdot dr = \frac{Q}{2\pi\varepsilon \cdot \ell} \int_{r_i}^{r_a} \frac{dr}{r} = \frac{Q}{2\pi\varepsilon \cdot \ell} \cdot \ln\left(\frac{r_a}{r_i}\right)$$

$$C = \frac{Q}{U} = \frac{2\pi\varepsilon \cdot \ell}{\ln\left(\frac{r_a}{r_i}\right)}$$

b) $$E_{(r)} = \frac{D_{(r)}}{\varepsilon} = \frac{Q}{2\pi\varepsilon \cdot \ell \cdot r} = \frac{U \cdot 2\pi\varepsilon \cdot \ell}{2\pi\varepsilon \cdot \ell \cdot r \cdot \ln\left(\frac{r_a}{r_i}\right)} = \frac{U}{r \cdot \ln\left(\frac{r_a}{r_i}\right)}$$

$$E_{(r=r_i)} = \frac{U}{r_i \cdot \ln\left(\frac{r_a}{r_i}\right)}$$

$$N = r_i \cdot \ln\left(\frac{r_a}{r_i}\right)$$

$$\frac{dN}{dr_i} = 1 \cdot \ln\left(\frac{r_a}{r_i}\right) + r_i \cdot \frac{1}{\frac{r_a}{r_i}} \cdot \left(\frac{r_a}{-r_i^2}\right) = \ln\left(\frac{r_a}{r_i}\right) - 1 \overset{!}{=} 0$$

$$\ln\left(\frac{r_a}{r_i}\right) = 1 \Rightarrow \frac{r_a}{r_i} = e \Rightarrow \underline{\underline{r_i = \frac{r_a}{e}}}$$

c) $I = \iint \vec{J} \cdot d\vec{A} = J_{(r)} \cdot 2\pi \cdot r \cdot \ell$

$$E_{(r)} = \frac{J_{(r)}}{\chi} = \frac{I}{2\pi\chi \cdot r \cdot \ell}$$

$$U_{(r)} = \int_{r_i}^{r_a} E_{(r)} \cdot dr = \frac{I}{2\pi\chi \cdot \ell}\int_{r_i}^{r_a} \frac{dr}{r} = \frac{I}{2\pi\chi \cdot \ell} \cdot \ln\left(\frac{r_a}{r_i}\right)$$

$$R = \frac{U}{I} = \frac{1}{2\pi\chi \cdot \ell} \cdot \ln\left(\frac{r_a}{r_i}\right)$$

d) $Q = D \cdot A = E \cdot \varepsilon \cdot A = \frac{J}{\chi} \cdot \varepsilon A = I \cdot \frac{\varepsilon}{\chi}$

$$\underline{\underline{R \cdot C = \frac{\varepsilon}{\chi}}}$$ für ebene Probleme

Lösung zur Übung 4.4.4.1/1

a) $C_{AB} = C_3 + \frac{C_1 C_2}{C_1 + C_2} = 3{,}5\mu F + \frac{2 \cdot 6}{8}\mu F = \underline{\underline{5\mu F}}$

b) $Q = C_1 \cdot U_1 = C_2 \cdot U_2$

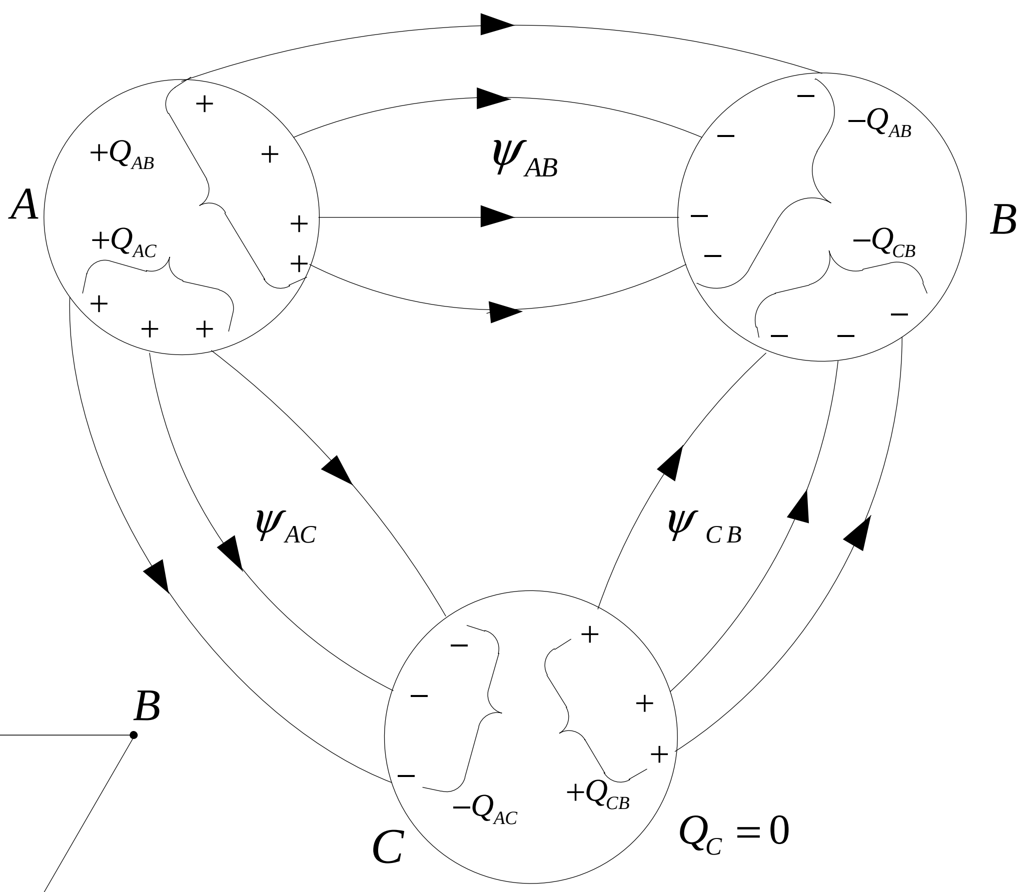
$$C_2 \cdot U_2 = C_1 \underbrace{(U_{AB} - U_2)}_{U_1}$$

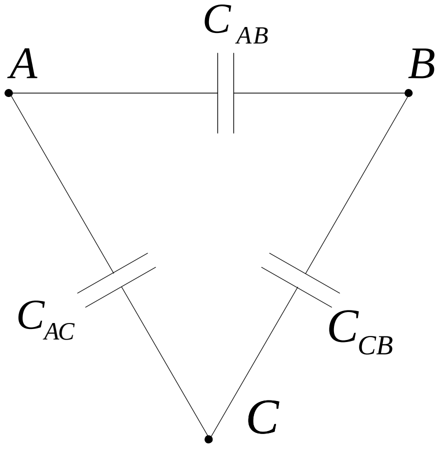
$$U_2 [C_1 + C_2] = U_{AB} \cdot C_1$$

$$U_2 = U_{AB} \cdot \frac{C_1}{C_1 + C_2} = 200V.\frac{2}{8} = \underline{\underline{50V}}$$

$$U_1 = U_{AB} - U_2 = (200 - 50)V = \underline{\underline{150V}}$$

Lösung zur Übung 4.4.4.2/1





Kugeln A, B wurden auf $+Q$ und $-Q$ aufgeladen

$$-Q = -Q_{AB} - Q_{CB}$$

$$Q = Q_{AB} + Q_{AC} = +Q_{AB} + Q_{CB} \Rightarrow Q_{AC} = Q_{CB}$$

$$C_{ges} = \frac{Q}{U_{AB}}$$

$$U_{AB} = U_{AC} + U_{CB} = \frac{Q_{AC}}{C_{AC}} + \frac{\overbrace{Q_{CB}}^{Q_{AC}}}{C_{CB}}$$

$$U_{AB} = \underbrace{Q_{AC}}_{Q-Q_{AB}}\left(\frac{1}{C_{AC}} + \frac{1}{C_{CB}}\right) = \left(\underbrace{Q}_{C_{ges}\cdot U_{AB}} - \underbrace{Q_{AB}}_{C_{AB}\cdot U_{AB}}\right)\left(\frac{1}{C_{AC}} + \frac{1}{C_{CB}}\right)$$

$$\overbrace{U_{AB}}^{1} = \left(C_{ges}\cdot U_{AB} - C_{AB}\cdot U_{AB}\right)\left(\frac{1}{C_{AC}} + \frac{1}{C_{CB}}\right)$$

$$1 = \left(C_{ges} - C_{AB}\right)\left(\frac{1}{C_{AC}} + \frac{1}{C_{CB}}\right)$$

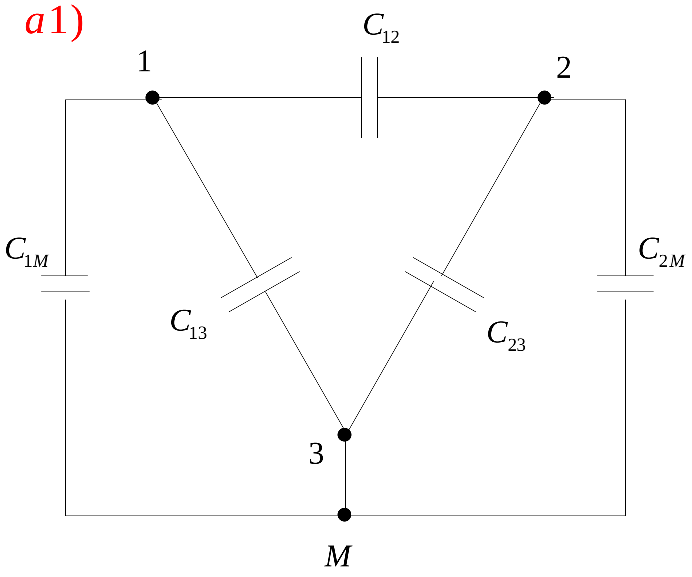
$$C_{ges} - C_{AB} = \frac{1}{\frac{1}{C_{AC}} + \frac{1}{C_{CB}}} = \frac{C_{AC}\cdot C_{CB}}{C_{AC} + C_{CB}}$$

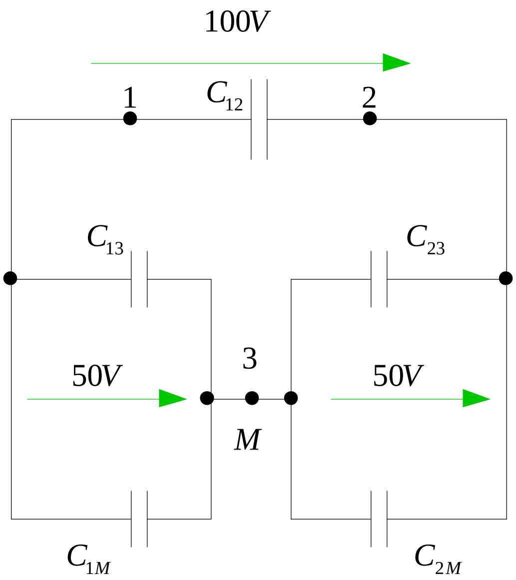
$$\underline{\underline{C_{ges} = C_{AB} + \frac{C_{AC}\cdot C_{CB}}{C_{AC} + C_{CB}}}}$$

A, B und C kann man als die Querschnitte eines Dreileitersystem auffassen

Lösung zur Übung 4.4.4.2/2

a1)

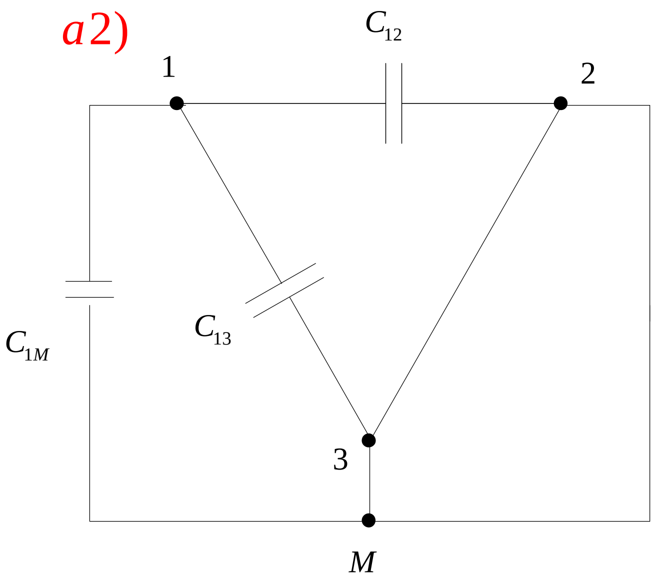


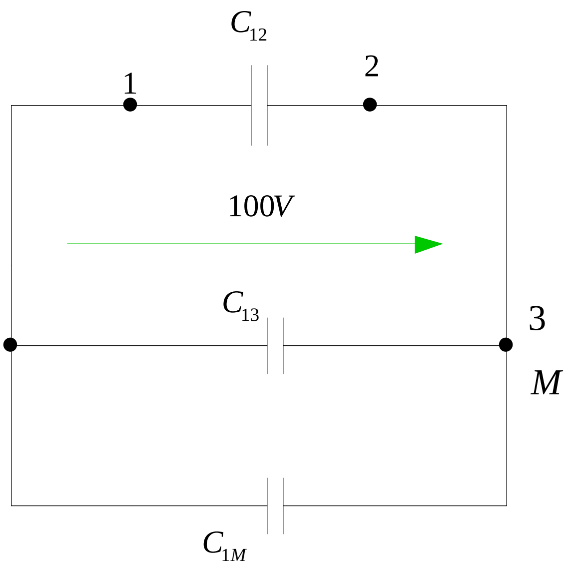


$$C_{B_{12}} = C_{12} + \frac{(C_{13} + C_{1M})(C_{23} + C_{2M})}{C_{13} + C_{1M} + C_{23} + C_{2M}}$$

$$C_{B_{12}} = 30\rho F + \frac{130 \cdot 130}{260}\rho F = \underline{\underline{95\rho F}}$$

a2)

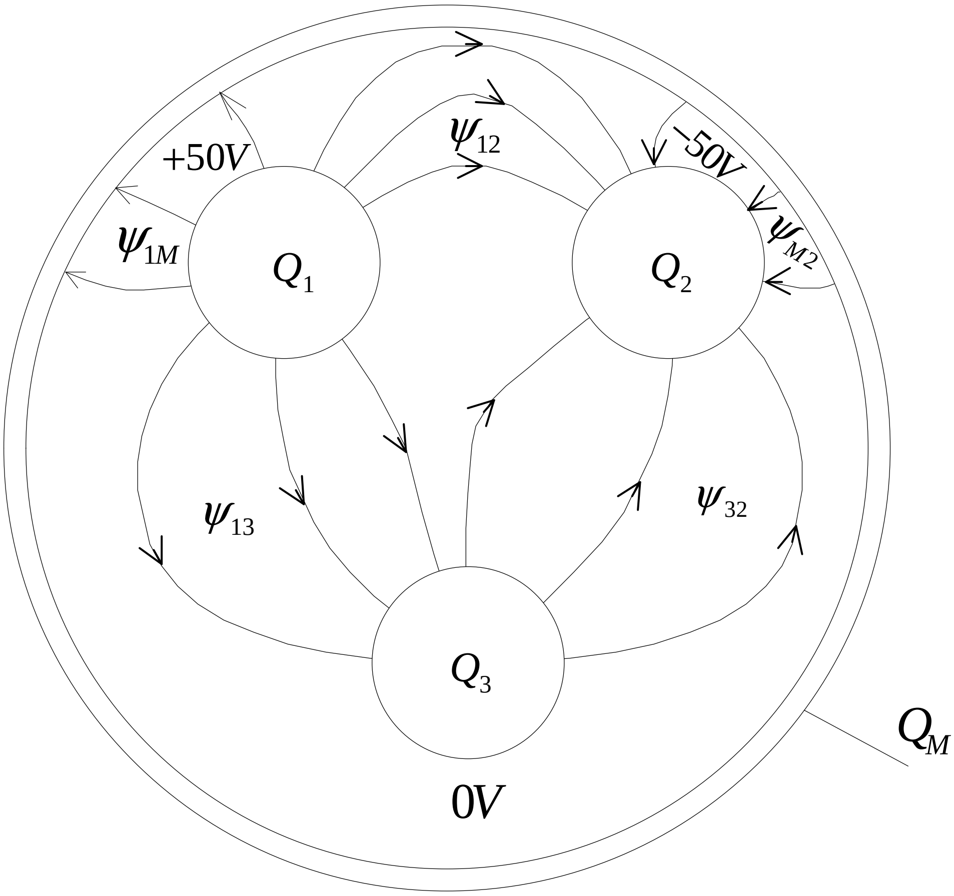




$$C_{B_{12}} = C_{12} + C_{13} + C_{1M}$$

$$C_{B_{12}} = 30\rho F + 30\rho F + 100\rho F = \underline{\underline{160\rho F}}$$

b1)



$$Q_1 = \psi_{12} + \psi_{13} + \psi_{1M} = C_{12}U_{12} + C_{13}U_{13} + C_{1M}U_{1M}$$

$$Q_1 = C_{12} \cdot 100V + C_{13} \cdot 50V + C_{1M} \cdot 50V$$

$$Q_1 = 30\rho F \cdot 100V + 50V\left(130\rho F\right) = \underline{\underline{9,5 \cdot 10^{-9}\,A\sec}}$$

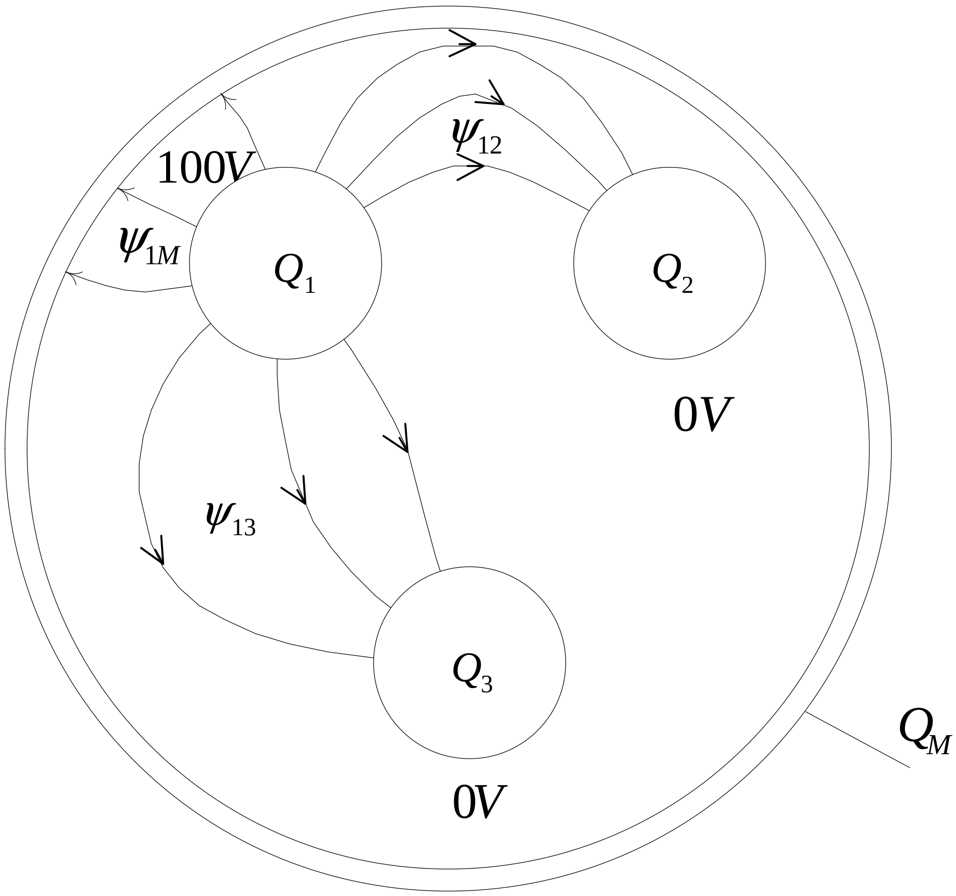
$$Q_2 = -\left[\psi_{12} + \psi_{32} + \psi_{M2}\right] = -\left[C_{12}U_{12} + C_{32}U_{32} + C_{M2}U_{M2}\right]$$

$$Q_2 = -\left(C_{12} \cdot 100V + C_{32} \cdot 50V + C_{M2} \cdot 50V\right) = \underline{\underline{-9,5 \cdot 10^{-9}\,A\sec}}$$

$$Q_3 = \psi_{32} - \psi_{13} = C_{32}U_{32} - \underbrace{C_{13}}_{C_{32}}U_{13} = C_{32}\left(\underbrace{U_{32}}_{50V} - \underbrace{U_{13}}_{50V}\right) = \underline{\underline{0}}$$

$$Q_M = \psi_{M2} - \psi_{1M} = C_{M2}U_{M2} - \underbrace{C_{1M}}_{C_{M2}}U_{M1} = C_{M2}\left(\underbrace{U_{M2}}_{50V} - \underbrace{U_{1M}}_{50V}\right) = \underline{\underline{0}}$$

b2)



$$Q_1 = \psi_{12} + \psi_{13} + \psi_{1M} = C_{12}U_{12} + C_{13}U_{13} + C_{1M}U_{1M}$$

$$Q_1 = C_{12} \cdot 100V + C_{13} \cdot 100V + C_{1M} \cdot 100V = 100V\left(C_{12} + C_{13} + C_{1M}\right)$$

$$Q_1 = 100V\left(30\rho F + 30\rho F + 100\rho F\right) = \underline{\underline{16 \cdot 10^{-9}\,Asec}}$$

$$Q_2 = -\psi_{12} = -C_{12}U_{12} = -30\rho F \cdot 100V = \underline{\underline{-3 \cdot 10^{-9}\,Asec}}$$

$$Q_3 = -\psi_{13} = -C_{13}U_{13} = -30\rho F \cdot 100V = \underline{\underline{-3 \cdot 10^{-9}\,Asec}}$$

$$Q_M = -\psi_{1M} = -C_{1M}U_{1M} = -100\rho F \cdot 100V = \underline{\underline{-10 \cdot 10^{-9}\,Asec}}$$

$$Kontrolle: \quad \sum Q = (16 - 3 - 3 - 10) \cdot 10^{-9}\,Asec = 0$$

Lösung zur Übung 4.4.6 / 1

a) $U = E_1 \cdot d_1 + E_2 \cdot d_2$

$D = \varepsilon \cdot E$ in beiden Dielektrika gleich $D_{n1} = D_{n2}$

$D = \varepsilon_0 \cdot \varepsilon_{r1} \cdot E_1 = \varepsilon_0 \cdot \varepsilon_{r2} \cdot E_2$

$$E_1 = \frac{D}{\varepsilon_0 \cdot \underbrace{\varepsilon_{r1}}_{1\ \text{für Luft}}} \qquad E_2 = \frac{D}{\varepsilon_0 \cdot \varepsilon_{r2}}$$

$$U = \frac{D}{\varepsilon_0}\left[\frac{d_1}{\varepsilon_{r1}} + \frac{d_2}{\varepsilon_{r2}}\right] \Rightarrow D = \frac{U \cdot \varepsilon_0}{\frac{d_1}{\varepsilon_{r1}} + \frac{d_2}{\varepsilon_{r2}}} = \frac{U \cdot \varepsilon_0}{\underbrace{\underbrace{d}}_{D_0 = \varepsilon_0 \cdot E_0}} \cdot \frac{d}{d_1 + \frac{d_2}{\varepsilon_{r2}}}$$

elektr. Erregung bei isotropem Dielektrikum mit $\varepsilon_r = 1$

$$D = \frac{2 \cdot 10^3 V \cdot 8{,}86 \cdot 10^{-12}}{\left(4 + \frac{6}{3}\right) \cdot 10^{-3} m} \frac{A\sec}{Vm} = \underline{\underline{2{,}95 \cdot 10^{-6} \frac{A\sec}{m^2}}}$$

$$E_1 = \frac{D}{\varepsilon_0} = \frac{2{,}95 \cdot 10^{-6} A\sec}{8{,}86 \cdot 10^{-12} m^2 \frac{A\sec}{Vm}} = \underline{\underline{3{,}33 \frac{kV}{cm}}}$$

$$E_2 = \frac{D}{\varepsilon_0 \cdot \varepsilon_{r2}} = \frac{E_1}{\varepsilon_{r2}} = \frac{3{,}33\ kV}{3\ \ cm} = \underline{\underline{1{,}11 \frac{kV}{cm}}}$$

$$Q = \oiint \vec{D} \cdot d\vec{A} = D \cdot A = 2{,}95 \cdot 10^{-6} \frac{A\sec}{m^2} \cdot 25 \cdot 10^{-4} m^2 = 7{,}38 \cdot 10^{-9} A\sec$$

b) für isotropes Dielektrikum mit $\varepsilon = \varepsilon_0 (\varepsilon_r = 1)$ gilt:

$$U_{(x)} = \int_0^x E \cdot dx = E \cdot x \qquad U_{(x=d)} = \int_0^{x=d} E \cdot dx = E \cdot d = 2kV$$

$$\Rightarrow E = \frac{2kV}{d} = \frac{2kV}{10mm} = \frac{2kV}{cm}$$

$$U_{(x)} = \frac{2kV}{cm} \cdot x$$

für geschichtetes Dielektrkum mit $\varepsilon_{r1} = 1,\ \varepsilon_{r2} = 3$ gilt :

$$0 \le x \le d_1$$

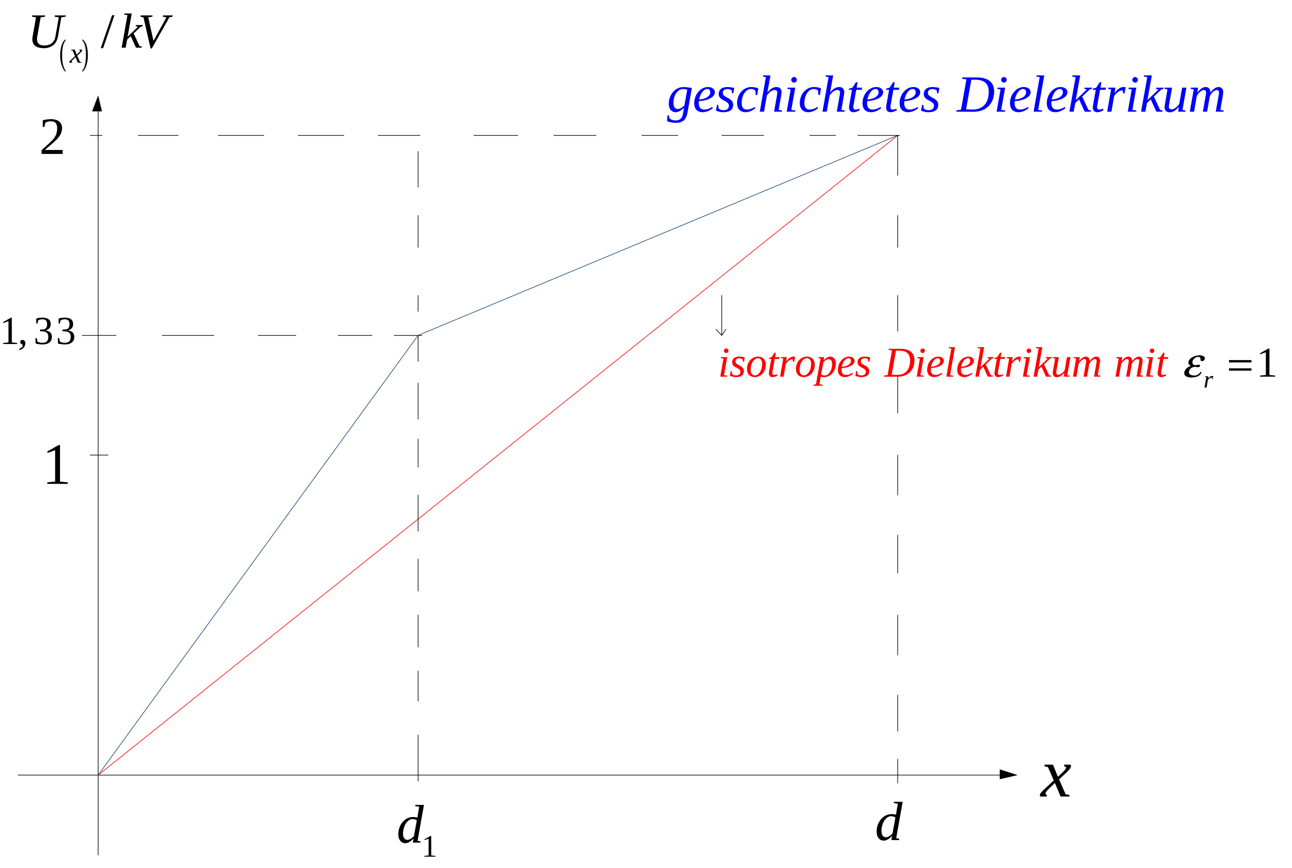
$$U_{(x)} = \int_0^x E_1 \cdot dx \ = E_1 \cdot x = 3,33 \frac{kV}{cm} \cdot x$$

$$U_1 = U_{(x=d_1)} = E_1 \cdot d_1 = 3,33 \frac{kV}{cm} \cdot 0,4cm = \underline{\underline{1,33kV}}$$

$$d_1 \le x \le d$$

$$U_{(x)} = U_{(x=d_1)} + \int_{d_1}^x E_2 \cdot dx \ = U_1 + E_2(x - d_1)$$

$$U_{(x=d)} = U_2 = U_1 + E_2(d - d_1) = 1,33kV + \frac{1,11kV \cdot 0,6cm}{cm} = \underline{\underline{2kV}}$$



$$c)\quad C=\frac{Q}{U}=\frac{\overbrace{D\cdot A}^{aus\ a)}}{U}=\frac{\varepsilon_0\cdot(\varepsilon_r)_{ers.}\cdot A}{d}$$

$$(\varepsilon_r)_{ers.}=\frac{\overbrace{D}^{aus\ a)}}{U}\cdot\frac{d}{\varepsilon_0}=\frac{U\cdot\varepsilon_0}{\frac{d_1}{\varepsilon_{r1}}+\frac{d_2}{\varepsilon_{r2}}}\cdot\frac{d}{U\cdot\varepsilon_0}=\frac{d}{\frac{d_1}{\varepsilon_{r1}}+\frac{d_2}{\varepsilon_{r2}}}=d\,\frac{\varepsilon_{r1}\cdot\varepsilon_{r2}}{d_1\varepsilon_{r2}+d_2\varepsilon_{r1}}$$

$$(\varepsilon_r)_{ers.}=\frac{10\cdot 1\cdot 3}{4\cdot 3+6\cdot 1}=\frac{30}{18}=1,\overline{666}$$

Zusatzbetrachtung: $C_{ers.}=\frac{\varepsilon_0\cdot A}{d}\cdot(\varepsilon_r)_{ers.}=\varepsilon_0\cdot A\frac{\varepsilon_{r1}\cdot\varepsilon_{r2}}{d_1\varepsilon_{r2}+d_2\varepsilon_{r1}}$

Reihenschaltung zweier Teilkapazitäten

$$C_{ers.}=\frac{C_1\cdot C_2}{C_1+C_2}=\frac{\frac{\varepsilon_0\cdot\varepsilon_{r1}\cdot A}{d_1}\cdot\frac{\varepsilon_0\cdot\varepsilon_{r2}\cdot A}{d_2}}{\frac{\varepsilon_0\cdot\varepsilon_{r1}\cdot A}{d_1}+\frac{\varepsilon_0\cdot\varepsilon_{r2}\cdot A}{d_2}}=\frac{\varepsilon_0\cdot A\cdot\varepsilon_{r1}\cdot\varepsilon_{r2}}{d_1\varepsilon_{r2}+d_2\varepsilon_{r1}}$$

Lösung zur Übung 4.4.6 / 2

$$a)\quad D_{n1}=D_{n2}=\varepsilon_1\cdot E_1=\varepsilon_2\cdot E_2\quad\Rightarrow\quad E_1=\frac{\varepsilon_2}{\varepsilon_1}\cdot E_2$$

$$U=\int\vec{E}\cdot d\vec{s}=E_1\cdot\frac{L}{2}+E_2\cdot\frac{L}{2}=\frac{\varepsilon_2}{\varepsilon_1}\cdot E_2\cdot\frac{L}{2}+E_2\cdot\frac{L}{2}$$

$$E_2\cdot\frac{L}{2}\left[\frac{\varepsilon_2}{\varepsilon_1}+1\right]=U$$

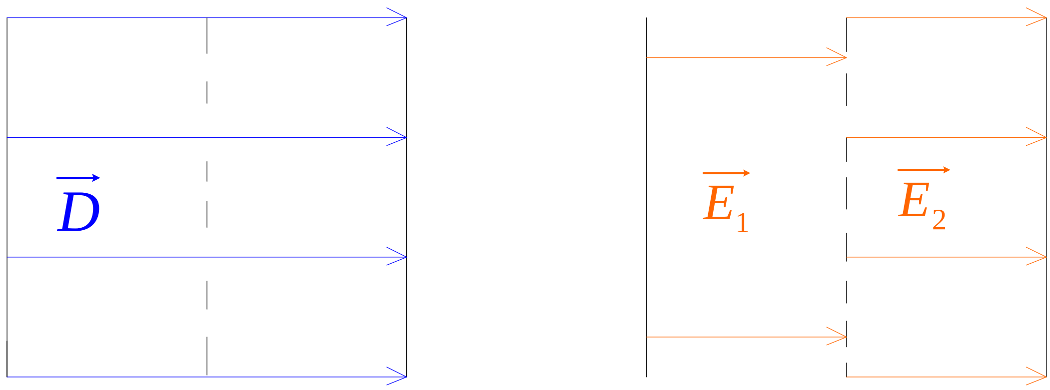
$$E_2=\frac{2U}{L}\cdot\frac{1}{\frac{\varepsilon_2}{\varepsilon_1}+1}=\frac{2U}{L}\cdot\frac{\varepsilon_1}{\varepsilon_1+\varepsilon_2}$$

$$E_2 = \frac{2U}{L} \cdot \frac{\varepsilon_{r1}}{\varepsilon_{r1} + \varepsilon_{r2}} = \frac{U}{L} \cdot \frac{4}{3}$$

$$U = E_1 \cdot \frac{L}{2} + \frac{L}{2} \cdot \frac{U}{L} \cdot \frac{4}{3}$$

$$E_1 \cdot \frac{L}{2} = U\left[1 - \frac{2}{3}\right] = \frac{1}{3}U \quad \Rightarrow \quad E_1 = \frac{2}{3} \cdot \frac{U}{L}$$

b)



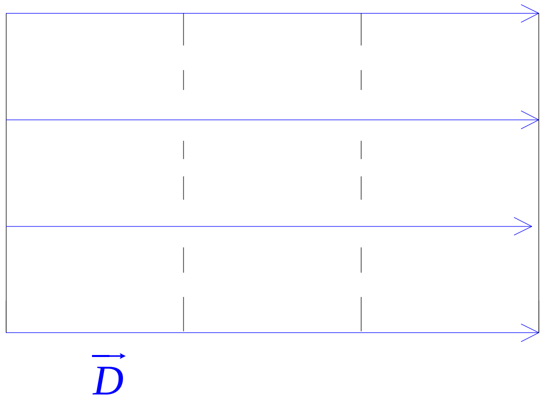
c) $$C = \frac{Q}{U} \qquad Q = \oiint \vec{D} \cdot d\vec{A} = D \cdot A_0$$

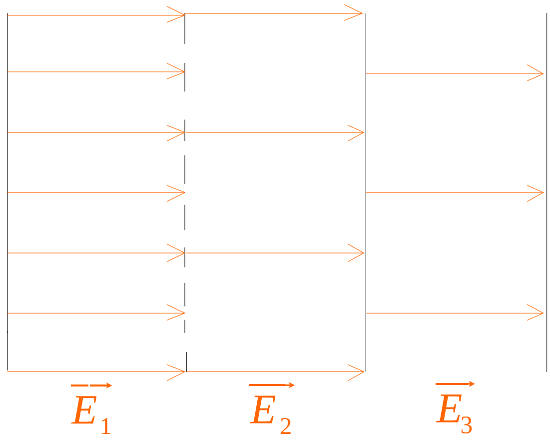
$$D = D_{n2} = \varepsilon_2 \cdot E_2 = \frac{2U}{L} \cdot \frac{\varepsilon_1 \varepsilon_2}{\varepsilon_1 + \varepsilon_2} = D_{n1} = \varepsilon_1 \cdot E_1$$

$$Q = \frac{2U}{L} \cdot \frac{\varepsilon_1 \varepsilon_2}{\varepsilon_1 + \varepsilon_2} \cdot A_0$$

$$C = \frac{2}{L} \cdot \frac{\varepsilon_1 \varepsilon_2}{\varepsilon_1 + \varepsilon_2} \cdot A_0 = \varepsilon_0 \frac{A_0}{L} \cdot \frac{2\varepsilon_{r1}\varepsilon_{r2}}{\varepsilon_{r1} + \varepsilon_{r2}} = \varepsilon_0 \frac{A_0}{L} \cdot \frac{8}{3}$$

Lösung zur Übung 4.4.6/3





$$D_{n1} = D_{n2} = D_{n3} = D \qquad D = \frac{Q}{A} \qquad aus\ Q = \oint\!\!\oint \vec{D} \cdot d\vec{A}$$

$$\vec{D} = \varepsilon \cdot \vec{E}$$

$$E_1 = \frac{D}{\varepsilon_1} \qquad E_2 = \frac{D}{\varepsilon_2} \qquad E_3 = \frac{D}{\varepsilon_3}$$

$$U = \int \vec{E} \cdot d\vec{s} = \int_0^{L/3} E_1 \cdot dx + \int_{L/3}^{2L/3} E_2 \cdot dx + \int_{2L/3}^{L} E_3 \cdot dx$$

$$= E_1 \cdot \frac{L}{3} + E_2 \cdot \frac{L}{3} + E_3 \cdot \frac{L}{3} = \frac{L}{3} \cdot D \left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} + \frac{1}{\varepsilon_3} \right)$$

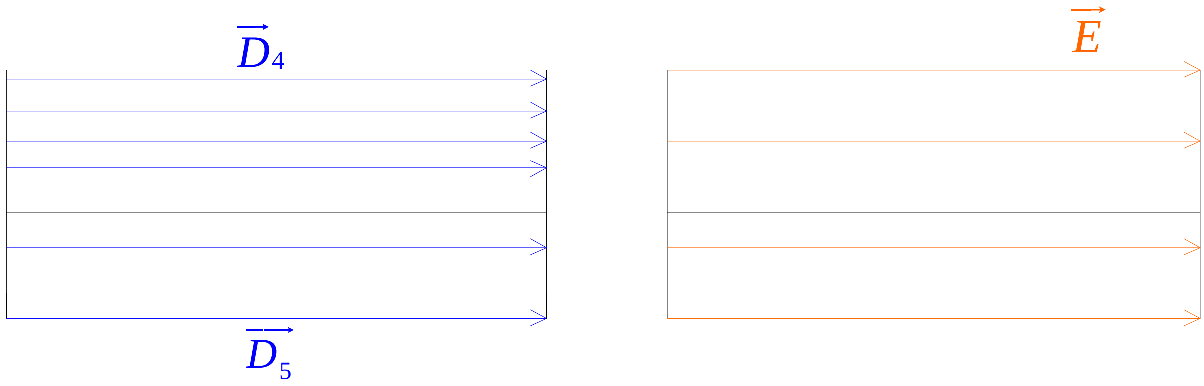
$$C_I = \frac{Q}{U} = \frac{\cancel{D} \cdot A}{\frac{L}{3} \cdot \cancel{D} \left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} + \frac{1}{\varepsilon_3} \right)} = \frac{3A}{L} \cdot \frac{1}{\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} + \frac{1}{\varepsilon_3} \right)}$$

$$E_{t4} = E_{t5} \Rightarrow E_4 = E_5 = \frac{U}{L}$$

$$D_4 = \varepsilon_4 \cdot E_4 = \varepsilon_4 \cdot \frac{U}{L} \qquad D_5 = \varepsilon_5 \cdot E_5 = \varepsilon_5 \cdot \frac{U}{L}$$

$$Q = \oint\!\!\oint \vec{D} \cdot d\vec{A} \Rightarrow Q_4 = D_4 \cdot \frac{A}{2} = \varepsilon_4 \cdot \frac{U}{L} \cdot \frac{A}{2}$$

$$Q_5 = D_5 \cdot \frac{A}{2} = \varepsilon_5 \cdot \frac{U}{L} \cdot \frac{A}{2}$$



$$Q_{ges} = Q_4 + Q_5 = \frac{U}{L} \cdot \frac{A}{2}\left(\varepsilon_4 + \varepsilon_5\right)$$

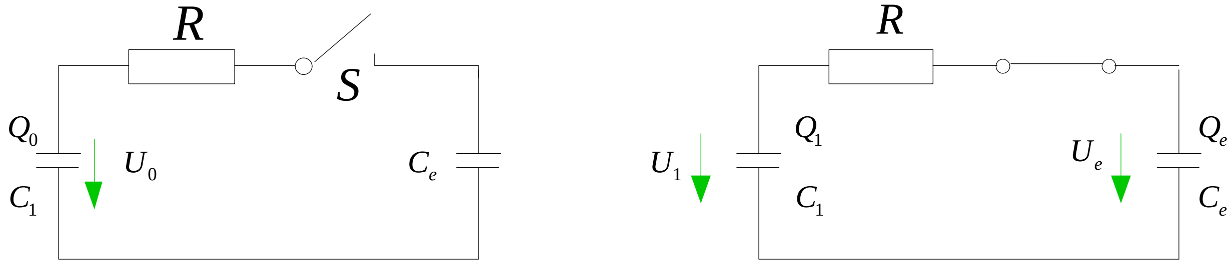
$$C_{II} = \frac{Q_{ges}}{U} = \underline{\underline{\frac{A}{2L}\left(\varepsilon_4 + \varepsilon_5\right)}}$$

$$C_{ges} = C_I + C_{II} = \underline{\underline{\frac{A}{L} \cdot \left[\frac{\varepsilon_4 + \varepsilon_5}{2} + \frac{3}{\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} + \frac{1}{\varepsilon_3}\right)}\right]}}$$

Lösung zur Übung 4 . 4 . 8 / 1

a) Zusammenfassen von C_2, C_3 und C_4: $C_e = C_4 + \frac{C_2 C_3}{C_2 + C_3}$ (1)

Kondensator C_1 besitzt die Ladung $Q_0 = C_1 \cdot U_0$ (2)



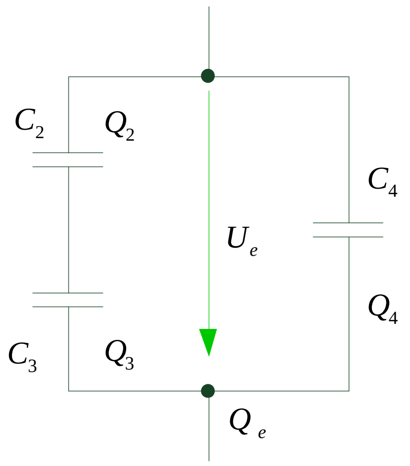
Die Gesamtladung Q_0 *hat sich aufgeteilt:* $Q_1 + Q_e = Q_0$ (3)

$i = 0: \Rightarrow U_1 = U_e$ (4)

$$U_1 = \frac{Q_1}{C_1} = U_e = \frac{Q_e}{C_e} \Rightarrow Q_1 = Q_e \cdot \frac{C_1}{C_e} \quad (5)$$

in (3) $$\underbrace{Q_e \cdot \frac{C_1}{C_e}}_{(5)} + Q_e = \underbrace{C_1 \cdot U_0}_{(2)} \Rightarrow Q_e = \frac{C_1 \cdot U_0}{1 + \dfrac{C_1}{C_e}} = \underline{\underline{\frac{C_1 \cdot C_e \cdot U_0}{C_1 + C_e}}} \quad (6)$$

$$\Rightarrow Q_1 = \underline{\underline{\frac{C_1^2}{C_1 + C_e} \cdot U_0}}_{mit\,(5)} \quad (7)$$



$$\left.\begin{array}{l} \text{Serienschaltung}: \quad Q_2 = Q_3 = Q_{2,3} \quad (8) \\ \text{Parallelschaltung}: Q_{2,3} + Q_4 = Q_e \quad (9) \end{array}\right\} U_e = \frac{Q_4}{C_4} = Q_{2,3} \cdot \frac{C_2 + C_3}{C_2 C_3} \quad (10)$$

mit (10) $$Q_4 = \underbrace{(Q_e - Q_4)}_{(9)} C_4 \cdot \frac{C_2 + C_3}{C_2 C_3}$$

$$Q_4 \left(1 + \frac{C_4}{C_2 C_3} \cdot (C_2 + C_3) \right) = Q_e \cdot \frac{C_4}{C_2 C_3} \cdot (C_2 + C_3)$$

$$Q_4 = \frac{Q_e \cdot \frac{C_4}{C_2 C_3} \cdot (C_2 + C_3)}{1 + \frac{C_4}{C_2 C_3} \cdot (C_2 + C_3)} = \frac{Q_e}{1 + \frac{C_2 C_3}{C_4 \cdot (C_2 + C_3)}} \quad (11)$$

$$Q_2 = Q_3 = Q_{2,3} = Q_e - Q_4 = Q_e \left(1 - \frac{\frac{C_4}{C_2 C_3} \cdot (C_2 + C_3)}{1 + \frac{C_4}{C_2 C_3} \cdot (C_2 + C_3)} \right) = \frac{Q_e}{1 + \frac{C_4}{C_2 C_3} \cdot (C_2 + C_3)} \quad (12)$$

b) *Energiebilanz*: *nach* (4.4.8/1) $W = \frac{1}{2} C \, U^2$

1. *Vor dem Schalten*: $W_1 = \frac{1}{2} C_1 \, U_0^{\,2}$ (13)

2. *Nach dem Schalten und der Umladung*:

$$W_2 = \underbrace{\frac{1}{2} C_1 \, U_1^{\,2}}_{A} + \underbrace{\frac{1}{2} C_e \, U_e^{\,2}}_{B} \quad (14)$$

$$A: \frac{1}{2} C_1 U_1^{\,2} = \frac{1}{2} C_1 \frac{Q_1^{\,2}}{C_1^{\,2}} = \frac{Q_1^{\,2}}{2C_1} = \frac{1}{2C_1} \underbrace{\left(\frac{C_1^{\,2}}{C_1 + C_e} \cdot U_0 \right)^2}_{nach\ (7)} = \frac{C_1^{\,3} \cdot U_0^{\,2}}{2 \cdot (C_1 + C_e)^2} \quad (15)$$

$$B: \frac{1}{2} C_e U_e^{\,2} = \frac{1}{2} C_e \frac{Q_e^{\,2}}{C_e^{\,2}} = \frac{Q_e^{\,2}}{2C_e} = \frac{1}{2C_e} \underbrace{\left(\frac{C_1 \cdot C_e \cdot U_0}{C_1 + C_e} \right)^2}_{nach\ (6)} = \frac{C_1^{\,2} \cdot C_e \cdot U_0^{\,2}}{2 \cdot (C_1 + C_e)^2} \quad (16)$$

$$nach\ (14)\ W_2 = \frac{C_1^{\,3} \cdot U_0^{\,2}}{2 \cdot (C_1 + C_e)^2} + \frac{C_1^{\,2} \cdot C_e \cdot U_0^{\,2}}{2 \cdot (C_1 + C_e)^2} = \frac{U_0^{\,2} \cdot C_1^{\,2}}{2} \cdot \frac{(C_1 + C_e)}{(C_1 + C_e)^2}$$

$$= \frac{U_0^{\,2}}{2} \cdot \frac{C_1^{\,2}}{C_1 + C_e} \quad (17)$$

Energiedifferenz $W_1 - W_2 = \frac{1}{2} C_1 U_0^2 - \frac{U_0^2}{2} \cdot \frac{C_1^2}{C_1 + C_e}$

$$= \frac{C_1 U_0^2}{2} \cdot \left(1 - \frac{C_1}{C_1 + C_e}\right) = \frac{C_1 U_0^2}{2} \cdot \frac{C_e}{C_1 + C_e}$$

ist im Widerstand R umgesetzt worden.

Zusatzfrage: Die Größe R kommt bei der Ableitung nicht vor.

Wo bleibt die Energiedifferenz, wenn R = 0?

$$[\tau] = [R] \cdot [C] = \frac{V}{A} \cdot \frac{A \sec}{V} = \sec$$

$\tau = 0 \Rightarrow$ Umladung geschieht in der Zeit = 0

$\tau \approx T = \frac{1}{f} \Rightarrow f \rightarrow \infty$ Abstrahlverluste

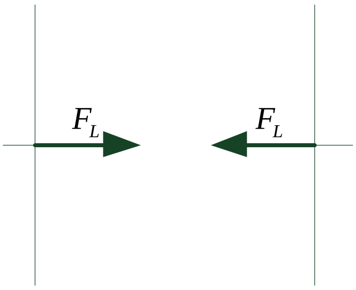
Lösung zur Übung 4.4.9/1

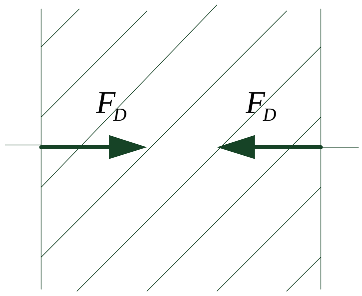
Die Anordnung kann als Reihenschaltung von zwei Kondensatoren aufgefasst werden.

$C_L \qquad C_D$

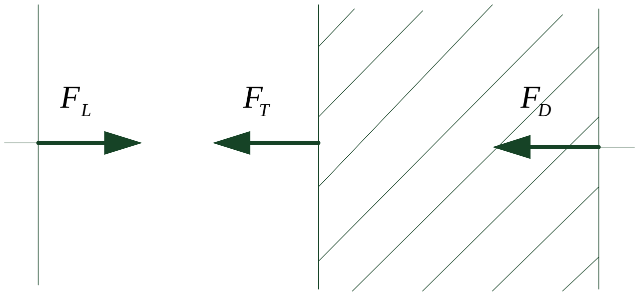
$$C_L = \frac{\varepsilon_0 A}{\ell_1} \qquad C_D = \frac{\varepsilon_0 \varepsilon_{r2} \cdot A}{\ell_2} \qquad L = \ell_1 + \ell_2 = const$$

Die anziehende Feldkraft auf die Platten ist in beiden Kondensatoren verschieden.





zusammengesetzt :



$$|F_T| = |F_L| - |F_D|$$

Energie nach (4.4.8 / 1) $W = \frac{1}{2} \cdot \frac{Q^2}{C}$

Energie im Dielektrikum : $W_D = \frac{1}{2} \cdot \frac{Q^2}{C_D} = \frac{1}{2} \cdot \frac{Q^2 \ell_2}{\varepsilon_0 \varepsilon_{r2} A} = \frac{1}{2} \cdot Q^2 \cdot \frac{L - \ell_1}{\varepsilon_0 \varepsilon_{r2} A}$

Energie in Luft : $W_L = \frac{1}{2} \cdot \frac{Q^2}{C_L} = \frac{1}{2} \cdot Q^2 \cdot \frac{\ell_1}{\varepsilon_0 A}$

$$|F| = \left| \frac{dW}{d\ell_1} \right| \qquad |F_L| = \left| \frac{dW_L}{d\ell_1} \right| = \left| \frac{1}{2} \cdot Q^2 \cdot \frac{1}{\varepsilon_0 A} \right| = \underline{\underline{\frac{1}{2} \cdot \frac{Q^2}{\varepsilon_0 A}}}$$

$$|F_D| = \left| \frac{dW_D}{d\ell_1} \right| = \left| -\frac{1}{2} \cdot Q^2 \cdot \frac{1}{\varepsilon_0 \varepsilon_{r2} A} \right| = \underline{\underline{\frac{1}{2} \cdot \frac{Q^2}{\varepsilon_0 \varepsilon_{r2} A}}}$$

$$|F_T| = |F_L| - |F_D| = \underline{\underline{\frac{1}{2} \cdot \frac{Q^2}{\varepsilon_0 A} \left[1 - \frac{1}{\varepsilon_{r2}} \right]}}$$

Lösung zur Übung 4.4.9 / 2

$$\left.\begin{array}{l} a)\ \varepsilon_2 > \varepsilon_1 \\ b)\ \varepsilon_2 < \varepsilon_1 \end{array}\right\}$$ Die Kraft wirkt in Richtung des kleineren ε

c) nein siehe Bsp. 4.4.9 / 1

Lösung zur Übung 4.4.9 / 3

a) $$C_0 = \frac{\varepsilon \cdot A}{d}$$

$$C_1 = \frac{\varepsilon \cdot A}{2d} = \frac{C_0}{2}$$

$U_B = const.$ $$U_B = \frac{Q_0}{C_0} = \frac{Q_1}{C_1}$$

$$Q_1 = Q_0 \cdot \frac{C_1}{C_0} = \frac{Q_0}{2} \Rightarrow Q_1 < Q_0$$

$\Rightarrow$ der Kondensator gibt Strom ab in der eingezeichneten Richtung

b) Die Batterie nimmt Energie auf.

Lösung zur Übung 4.4.9 / 4

a) $Q = const.$ $$C_1 = \frac{\varepsilon_0 \varepsilon_r A}{d}$$

ohne Dielektrikum $$C_2 = \frac{\varepsilon_0 A}{d} < C_1$$

$$\left.\begin{array}{l} W_1 = \frac{1}{2} \cdot \frac{Q^2}{C_1} \\ W_2 = \frac{1}{2} \cdot \frac{Q^2}{C_2} \end{array}\right\} C_2 < C_1 \Rightarrow W_2 > W_1$$

Die Energie wird größer

b) $U = const.$

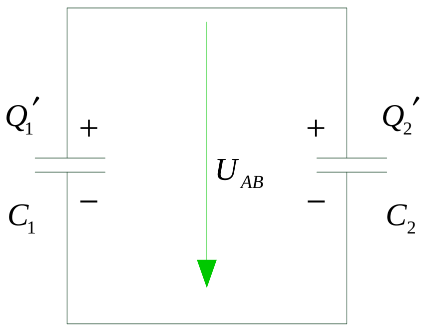
$$\left.\begin{aligned} W_1 &= \frac{1}{2} C_1 U^2 \\ W_2 &= \frac{1}{2} C_2 U^2 \end{aligned}\right\} C_2 < C_1 \Rightarrow W_2 < W_1$$

Die Energie wird kleiner

Lösung zur Übung 4.4.9/5

$$Q_1 = C_1 U_1 = 10^{-8} \frac{A\sec}{V} \cdot 10^2 V = 10^{-6} A\sec$$

$$Q_2 = C_2 U_2 = 5 \cdot 10^{-8} \frac{A\sec}{V} \cdot 2 \cdot 10^1 V = 10^{-6} A\sec$$



Parallelschaltung: $C'_{ges} = C_1 + C_2$

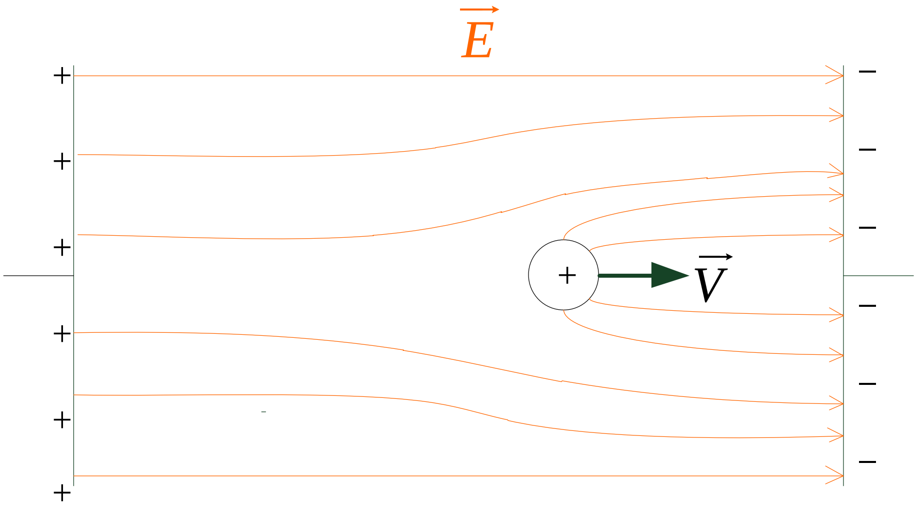
$$Q'_{ges} = Q'_1 + Q'_2 = Q_1 - Q_2$$

$$U_{AB} = \frac{Q'_{ges}}{C'_{ges}} = \frac{Q_1 - Q_2}{C_1 + C_2} = \frac{C_1 U_1 - C_2 U_2}{C_1 + C_2} = \underline{\underline{0}}$$

Lösung zur Übung 4.4.9/6

a) Auf die geladene Kugel wird vom Feld eine Kraft ausgeübt:
Feldenergie wird in Bewegungsenergie umgesetzt
$\Rightarrow$ Feldenergie wird geringer

b) Feldenergie bleibt gleich, da Spannungsquelle "verlorene" Energie nachliefert



Feldverzerrung $\Rightarrow$ Kapazität ändert sich

Lösung zur Übung 4 . 5 / 1

a) Strom durch R_1 : I_1 von A nach B (C_1 wird auf U aufgeladen)
Strom durch R_2 : I_2 von C nach B (C_2 wird auf null entladen)
Strom durch R_3 : I_3 von B nach D ($I_3 = I_1 + I_2$)

b) Da der Kondensator C_1 auf die volle Spg. U aufgeladen ist, liegt nach Öffnen des Schalters S an C_2 keine Spg.. Es können deswegen auch keine Ströme fließen: $I_1 = I_2 = I_3 = 0$

c) c1) $W_1 = \frac{1}{2}U^2 \cdot \frac{C_1 \cdot C_2}{C_1 + C_2}$ Serienschaltung

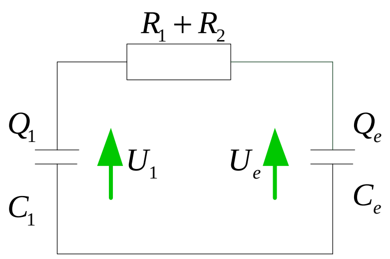
c2) $W_2 = \frac{1}{2}U^2 C_1$ weil $U_{C_2} = 0$

c3) $W_3 = \frac{1}{2}U^2 C_1$ kein Energiefluss nach dem Schalten

Lösung zur Übung 4 . 5 / 2

a) Kondensator C_1 besitzt die Ladung $Q_1' = C_1 \cdot U_1' = 10^{-6}\frac{Asec}{V} \cdot 10V = 10^{-5} Asec$

Zusammenfassen von C_2, C_3 und C_4: $C_e = \frac{C_2(C_3+C_4)}{C_2+C_3+C_4} = \frac{2\cdot 8}{10}\mu F = 1{,}6\mu F$



$U_1 = U_e$ weil $i = 0$

$\frac{Q_1}{C_1} = \frac{Q_e}{C_e} \qquad Q_1' = Q_1 + Q_e \Rightarrow Q_e = Q_1' - Q_1$

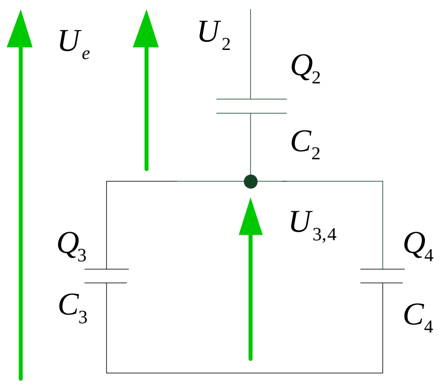
$Q_1 = Q_e \cdot \frac{C_1}{C_e} = (Q_1' - Q_1)\cdot \frac{C_1}{C_e}$

$Q_1\left(1 + \frac{C_1}{C_e}\right) = Q_1' \cdot \frac{C_1}{C_e}$

$$Q_1 = \frac{Q_1' \cdot \frac{C_1}{C_e}}{1+\frac{C_1}{C_e}} = \frac{Q_1'}{1+\frac{C_e}{C_1}} = \frac{10^{-5} A\sec}{1+\frac{1,6}{1}} = 3,846\cdot 10^{-6} A\sec$$

$$Q_e = Q_1' - Q_1 = \left(10^{-5} - 0,3846\cdot 10^{-5}\right) A\sec = 0,6154\cdot 10^{-5} A\sec$$

$$U_1 = \frac{Q_1}{C_1} = \frac{3,846\cdot 10^{-6}\ A\sec}{10^{-6}\ \frac{A\sec}{V}} = \underline{\underline{3,846\ V}}$$



Parallelschaltung $Q_p = Q_3 + Q_4$

Serienschaltung (alle Ladungen gleich)

$Q_2 = Q_p = Q_3 + Q_4$

$Q_e = Q_2 = Q_3 + Q_4$

$Q_2 = Q_e = 0,6154\cdot 10^{-5} A\sec$

$$U_2 = \frac{Q_2}{C_2} = \frac{0,6154\cdot 10^{-5}\ A\sec}{2\cdot 10^{-6}\ \frac{A\sec}{V}} = \underline{\underline{3,077\ V}}$$

$U_e = U_1 = 3,846\ V$

$U_e = U_2 + U_{3,4} \Rightarrow U_{3,4} = U_e - U_2 = 3,846\ V - 3,077\ V = \underline{\underline{0,769\ V}}$

$b)$ *Energiebilanz:*

1. *Vor dem Schalten:* $W_I = \frac{1}{2} C_1 U_1'^2 = \frac{1}{2} \cdot 10^{-6} \frac{A\sec}{V} \cdot 10^2 V^2 = 5 \cdot 10^{-5} VA\sec$

2. *Nach dem Schalten:* $W_{II} = \frac{1}{2} C_1 U_1^2 + \frac{1}{2} C_e U_e^2$

$$W_{II} = \frac{1}{2} \cdot 10^{-6} A\sec \cdot (3{,}846\ V)^2 + \frac{1}{2} \cdot 1{,}6 \cdot 10^{-6} \cdot (3{,}846\ )^2 V$$

$$W_{II} = \frac{(3{,}846\ V)^2}{2} \cdot 2{,}6 \cdot 10^{-6} A\sec = 1{,}923 \cdot 10^{-5} VA\sec$$

$$\Delta W = W_I - W_{II} = 3{,}077 \cdot 10^{-5} VA\sec$$

$$P = I^2 \cdot R \Rightarrow P \sim R$$

$$W \sim P \sim R$$

$$\frac{W_1}{W_2} = \frac{R_1}{R_2} = \frac{1}{4} \Rightarrow W_2 = 4W_1$$

$$\Delta W = W_1 + W_2 = W_1 + 4W_1 = 5W_1$$

$$W_1 = \frac{\Delta W}{5} = \underline{\underline{0{,}6154 \cdot 10^{-5} VA\sec}}$$

$$W_2 = 4W_1 = \underline{\underline{2{,}4616 \cdot 10^{-5} VA\sec}}$$